Cellular Automata with Memory

Ramón Alonso-Sanz (UPM,Spain)
Margarita Martín (UCM,Spain)
ramon.alonso@upm.es

Standard CA

Standard Cellular Automata (CA) are ahistoric (memoryless): i.e., the new state of a cell depends on the neighborhood configuration only at the preceding time step.

$$\sigma_i^{(T+1)} = \phi\big(\sigma_j^{(T)} \in \mathcal{N}_i\big)$$

$$\boxed{\textbf{CA with memory in cells}}$$

$$\sigma_i^{(T+1)} = \phi\big(s_j^{(T)} \in \mathcal{N}_i\big)$$

$$s_j^{(T)} = s\big(\sigma_j^{(1)}, \ldots, \sigma_j^{(T-1)}, \sigma_j^{(T)}\big)$$

This contribution considers an extension to the standard framework of CA by implementing memory capabilities in cells. Thus in CA with memory here: while the update rules of the CA remain unaltered, historic memory of all past iterations is retained by featuring each cell (and link) by a summary of its past states.

Example: $$s_i^{(T)} = \underline{mode}(\sigma_i^{(1)}, \ldots, \sigma_i^{(T)})$$

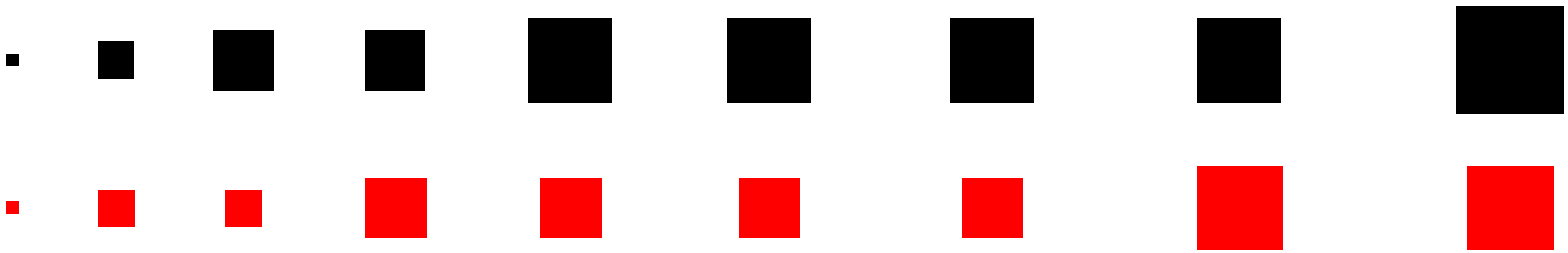
$$s_i^{(T)} = \sigma_i^{(T)} \textbf{ in case of a tie: } card\{1\} = card\{0\}$$

Ex.- *Speed of light*: cell alive if any cell in its neighb. alive.

Ahistoric

σ

Mode (Full) memory



σ

s

INERTIAL EFFECT

Weighted memory

Unlimited trailing

$$m_i^{(T)}(\sigma_i^{(1)}, \ldots, \sigma_i^{(T)}) = \frac{\sigma_i^{(T)} + \sum_{t=1}^{T-1} \alpha^{T-t} \sigma_i^{(t)}}{1 + \sum_{t=1}^{T-1} \alpha^{T-t}} \equiv \frac{\omega_i^{(T)}}{\Omega(T)}$$

$$0 \le \alpha \le 1$$

The choice of the memory factor α simulates the long-term or remnant memory effect: the limit case $\alpha = 1$ corresponds memory with equally weighted records ($full$ memory model, $mode$ if $k = 2$), whereas $\alpha << 1$ intensifies the contribution of the most recent states and diminishes the contribution of the past ones ($short$ type memory). The choice $\alpha = 0$ leads to the ahistoric model.

If $\sigma \in \{0, 1\}$, the rounded weighted mean state (s) will be obtained by comparing the weighted mean (m) to 0.5, so that :

$$s_i^{(T)} = \begin{cases} 1 & if \quad m_i^{(T)} > 0.5 \\ \sigma_i^{(T)} & if \quad m_i^{(T)} = 0.5 \\ 0 & if \quad m_i^{(T)} < 0.5 \end{cases} .$$

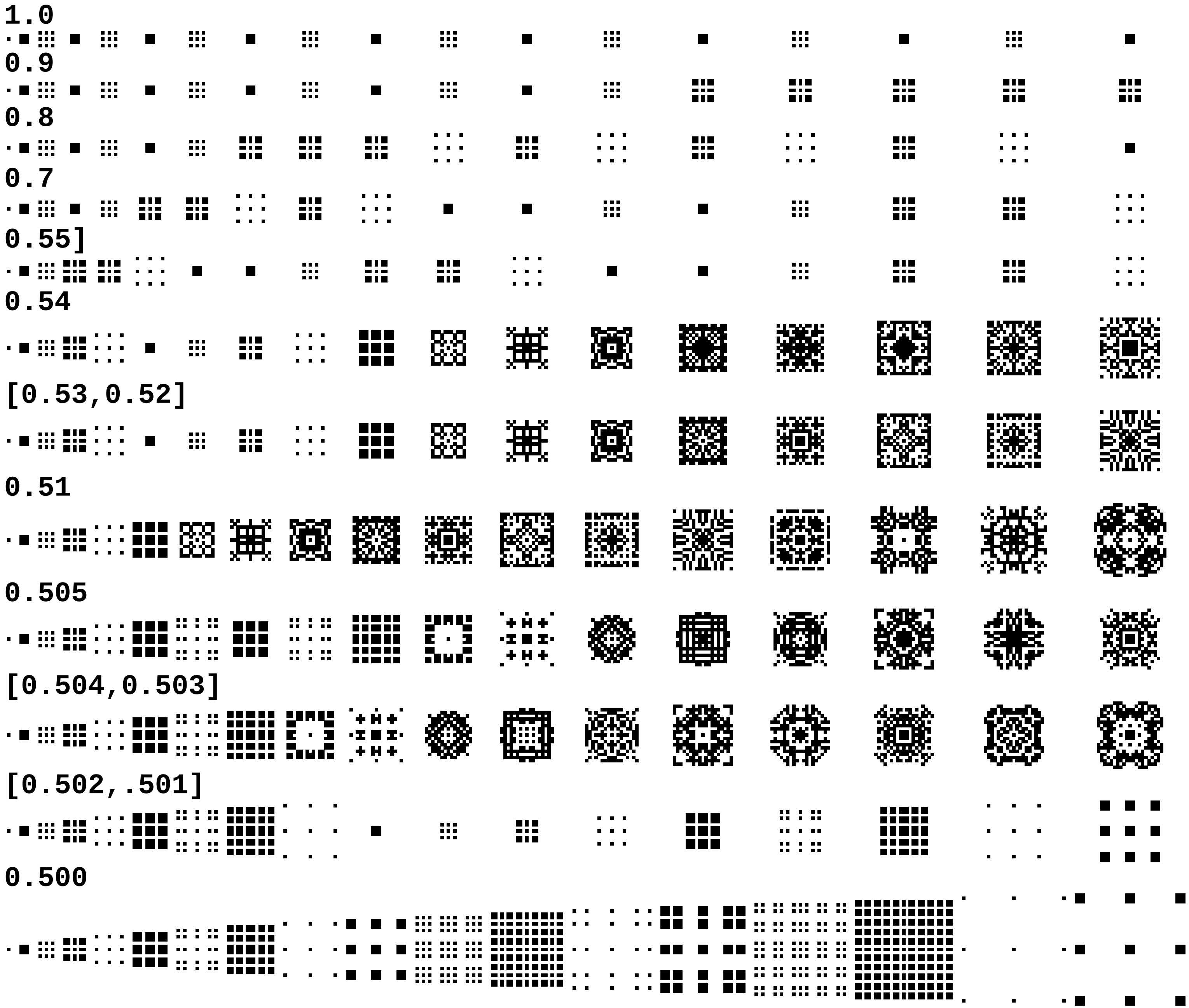
$$s_i^{(1)} = \sigma_i^{(1)} \ , \ s_i^{(2)} = \sigma_i^{(2)}$$

Implementation

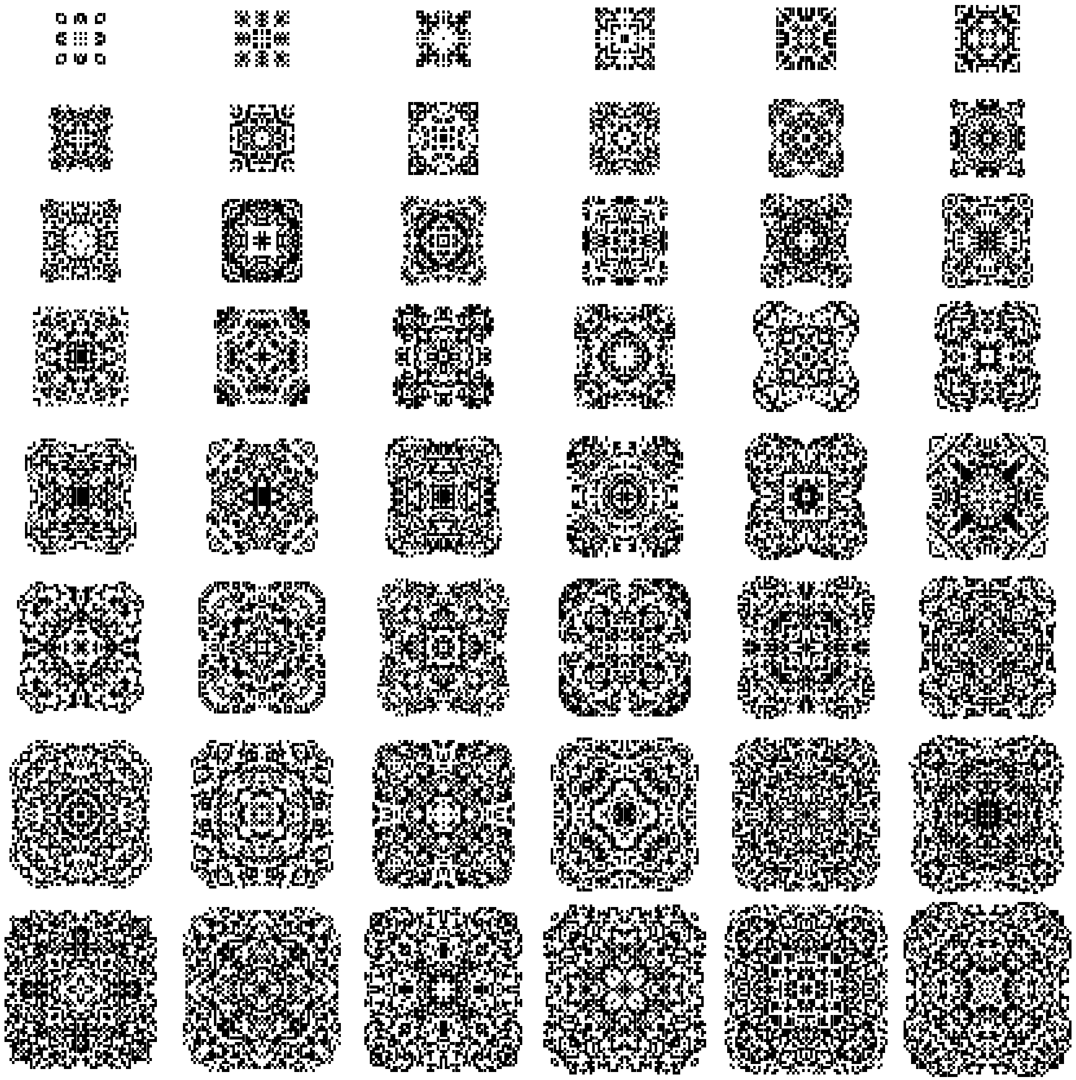
$$\omega_i^{(T)} = \alpha \omega_i^{(T-1)} + \sigma_i^{(T)} \quad \rightarrow \quad \{\sigma_i^{(t)}\} \textbf{ NO NEEDED}$$

$k = 2 :$ **α-MEMORY EFFECTIVE if $\alpha > 0.5$**

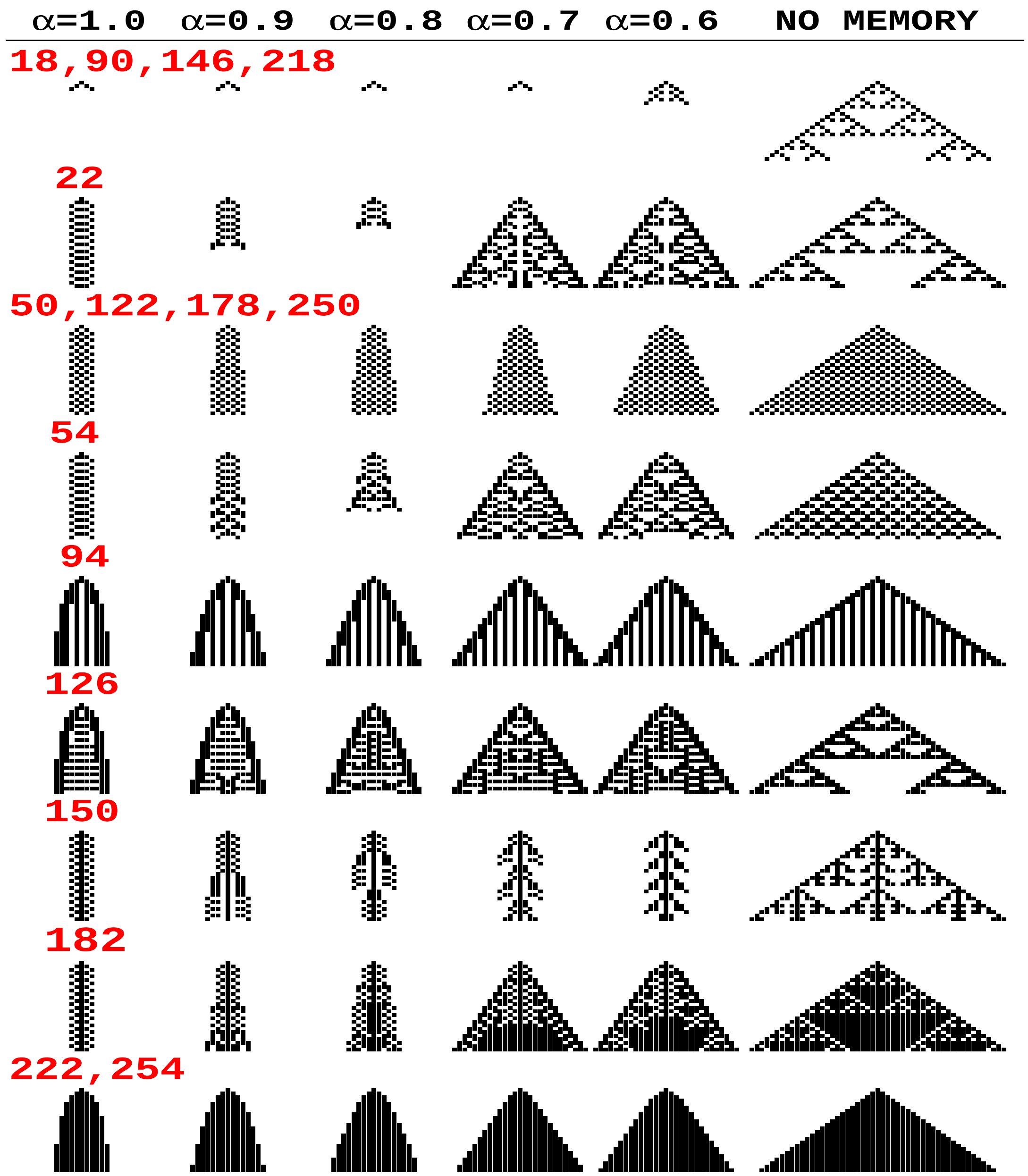
The 2D PARITY rule with Memory. Moore N. [17]



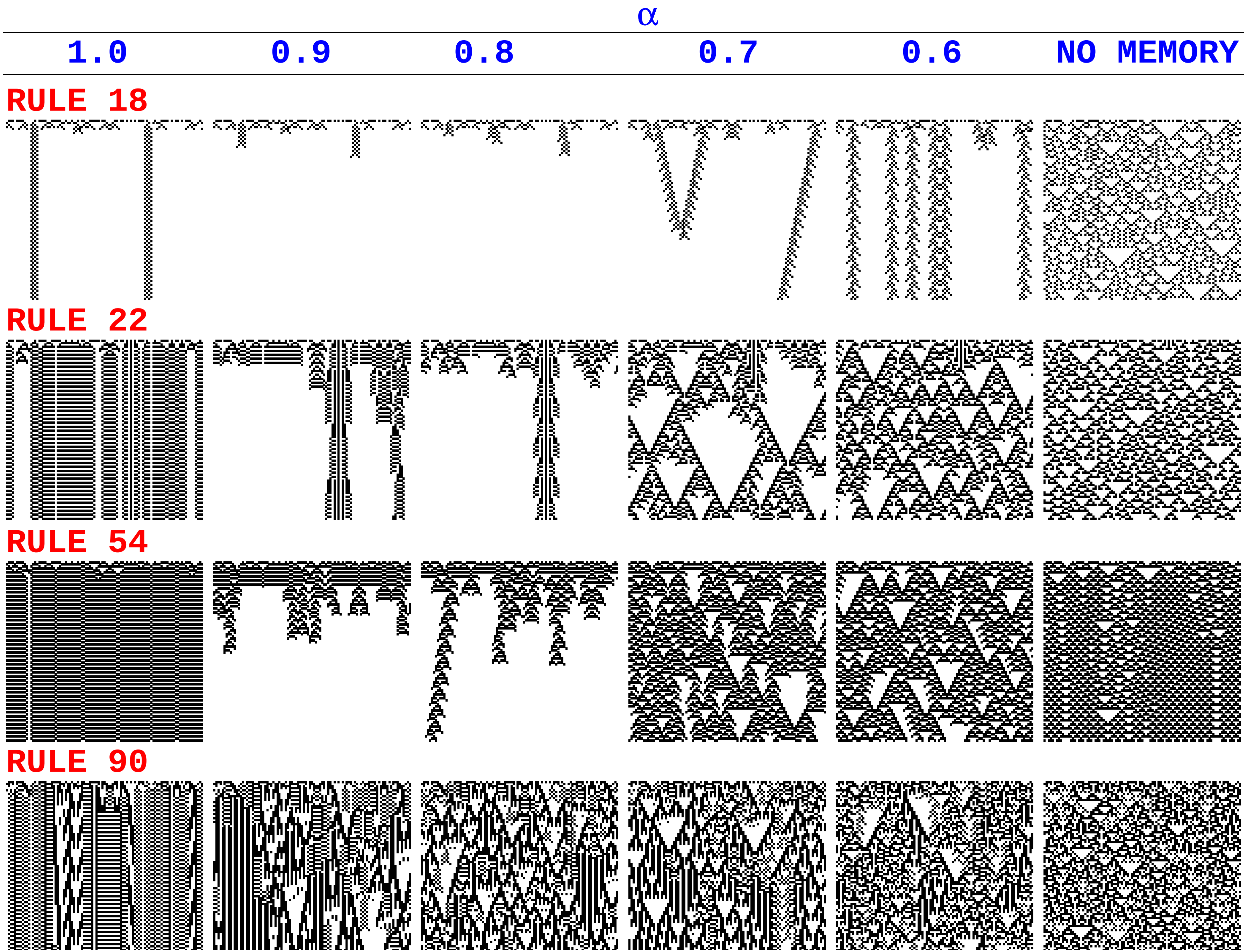
PARITY RULE α=0.501 T=19-66



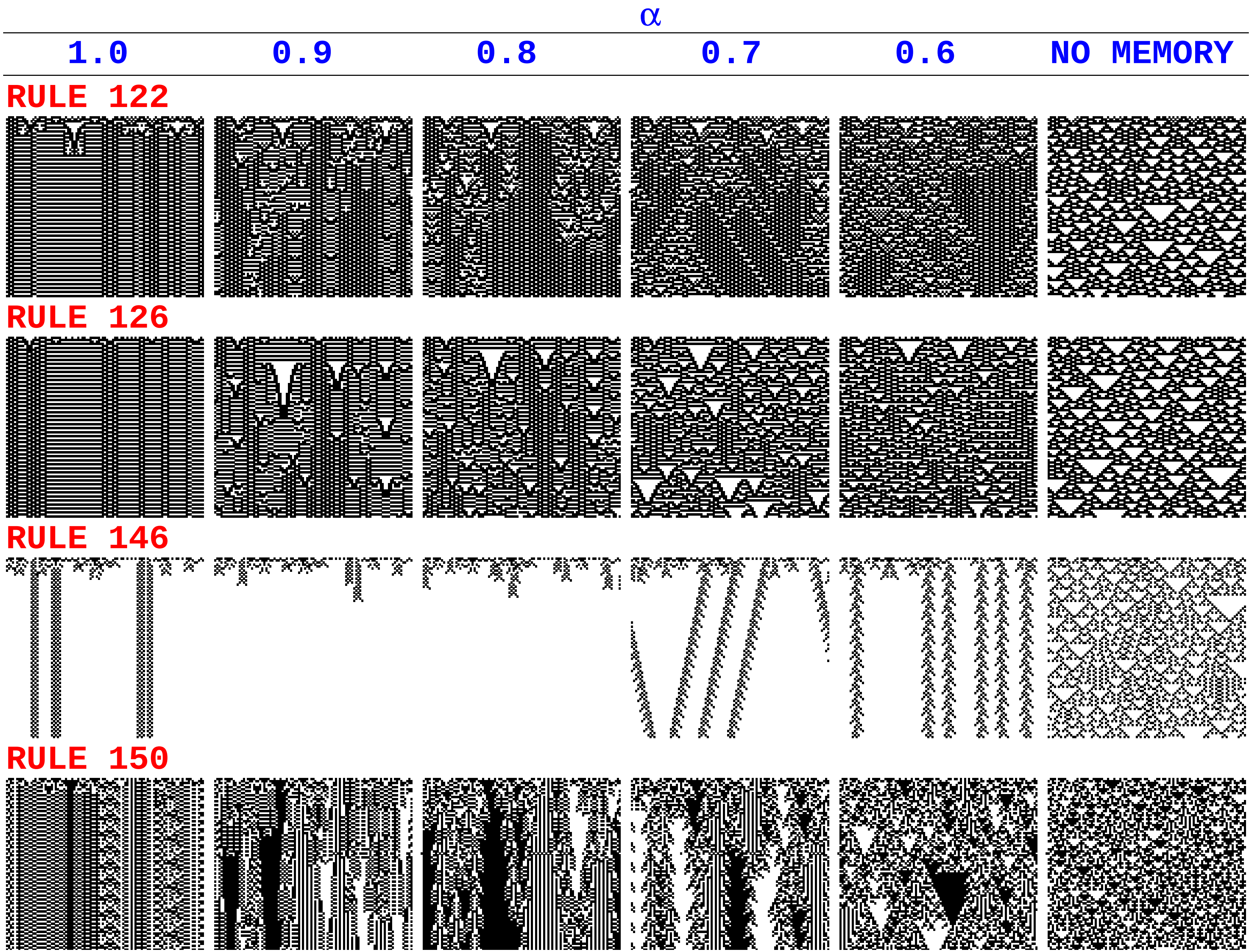
Elementary, Legal Rules with Memory [13]



Elementary, Legal Rules with Memory [13]



Elementary, Legal Rules with Memory [15]



Average-like memory mechanisms

$$m_i^{(T)} = \frac{\sum_{t=1}^{T} \delta(t)\sigma_i^{(t)}}{\sum_{t=1}^{T} \delta(t)} \equiv \frac{\omega_i^{(T)}}{\Omega(T)}$$

- **exponential :** $\delta(t) = e^{-\beta(T-t)} \qquad \beta \in \mathbb{R}^+$

$$\alpha = e^{-\beta}$$

- **inverse:** $\delta(t) = \alpha^{t-1} \qquad \delta(t) > \delta(t+1)$
- **integer-based (*à la* CA):** $c \in \mathbb{N}$ [**14**]

$$\delta(t) = t^c \;\rightarrow\; \omega_i^{(T)} = \omega_i^{(T-1)} + T^c \sigma_i^{(T)}$$

$$\delta(t) = c^t \;\rightarrow\; \omega_i^{(T)} = \omega_i^{(T-1)} + c^T \sigma_i^{(T)}$$

<u>Limited trailing</u> (τ states)

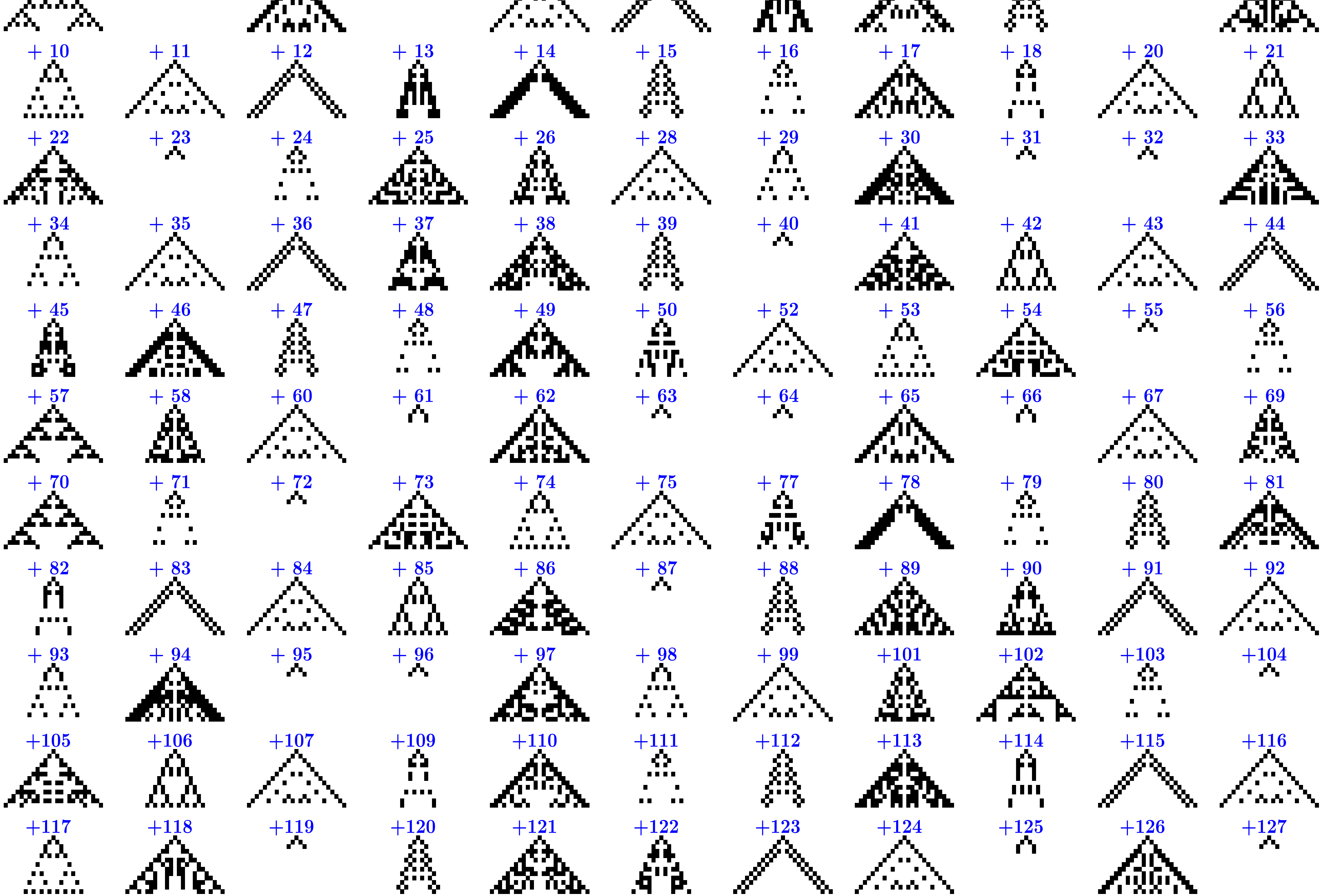
- **α-memory**

$$m_i^{(T)}(\sigma_i^{(T-\tau+1)},\ldots,\sigma_i^{(T)}) = \frac{\displaystyle\sum_{t=\top}^{T}\delta(t)\sigma_i^{(t)}}{\displaystyle\sum_{t=\top}^{T}\delta(t)} \qquad \top = \max(1, T-\tau+1)$$

- **Elementary Rules as Memory ($\tau = 3$):**

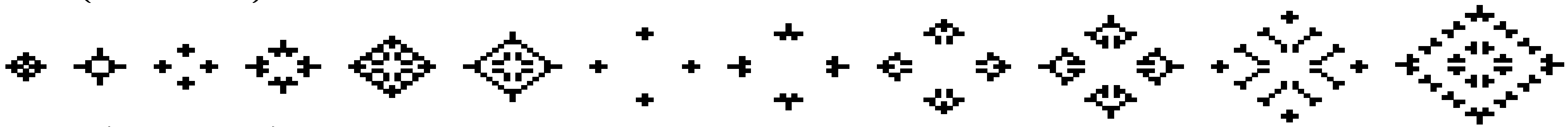
$$s_i^{(T)} = \phi\big(\sigma_i^{(T-2)}, \sigma_i^{(T)}, \sigma_i^{(T-1)}\big) \qquad [\mathbf{5}]$$

$$\textbf{e.g.:} \quad s_i^{(T)} = mode\big(\sigma_i^{(T-2)}, \sigma_i^{(T)}, \sigma_i^{(T-1)}\big) \equiv \textbf{ER 232} \qquad [\mathbf{8}]$$

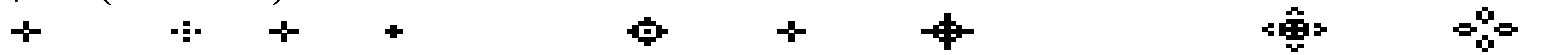

+ 10
+ 11
+ 12
+ 13
+ 14
+ 15
+ 16
+ 17
+ 18
+ 20
+ 21
+ 22
+ 23
+ 24
+ 25
+ 26
+ 28
+ 29
+ 30
+ 31
+ 32
+ 33
+ 34
+ 35
+ 36
+ 37
+ 38
+ 39
+ 40
+ 41
+ 42
+ 43
+ 44
+ 45
+ 46
+ 47
+ 48
+ 49
+ 50
+ 52
+ 53
+ 54
+ 55
+ 56
+ 57
+ 58
+ 60
+ 61
+ 62
+ 63
+ 64
+ 65
+ 66
+ 67
+ 69
+ 70
+ 71
+ 72
+ 73
+ 74
+ 75
+ 77
+ 78
+ 79
+ 80
+ 81
+ 82
+ 83
+ 84
+ 85
+ 86
+ 87
+ 88
+ 89
+ 90
+ 91
+ 92
+ 93
+ 94
+ 95
+ 96
+ 97
+ 98
+ 99
+101
+102
+103
+104
+105
+106
+107
+109
+110
+111
+112
+113
+114
+115
+116
+117
+118
+119
+120
+121
+122
+123
+124
+125
+126
+127

The Parity rule with Elementary Rules as Memory. T=4-15. vNN

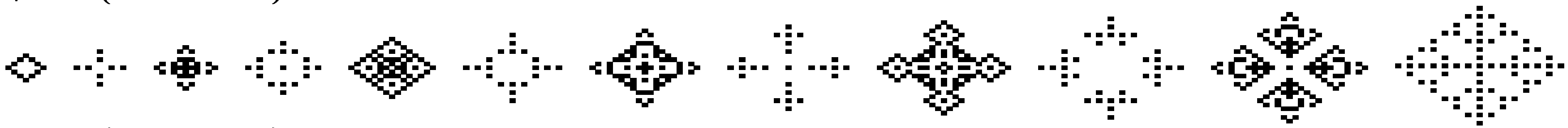
+ 4 (00000100)



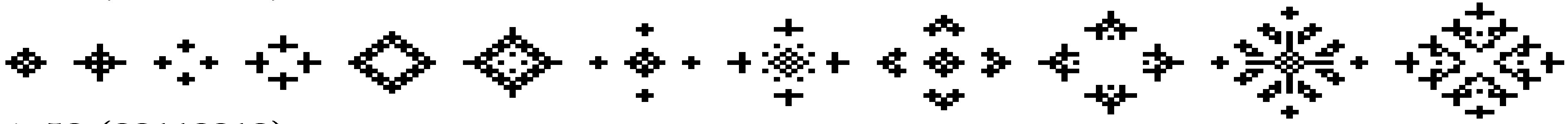
+ 18 (00010010)



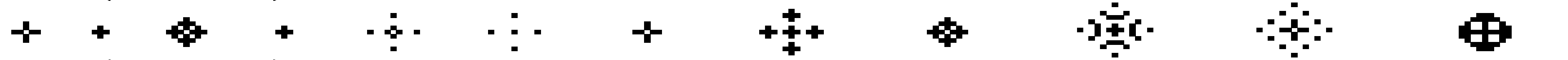
+ 22 (00010110)



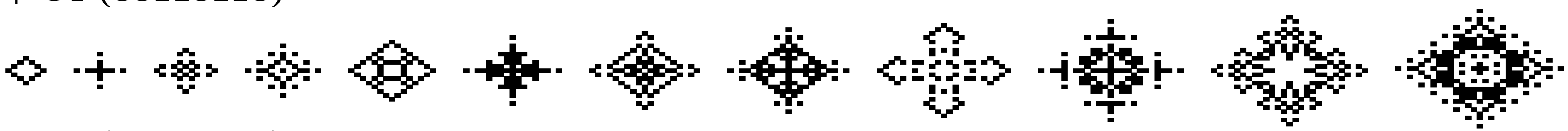
+ 36 (00100100)



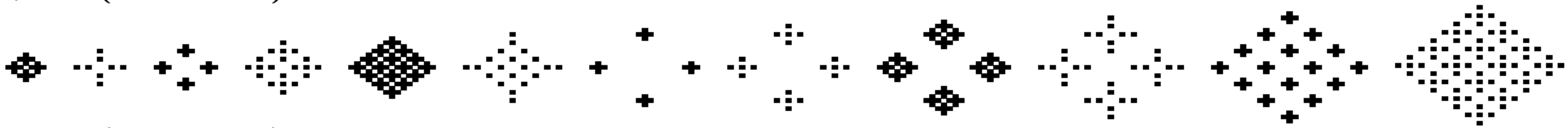
+ 50 (00110010)



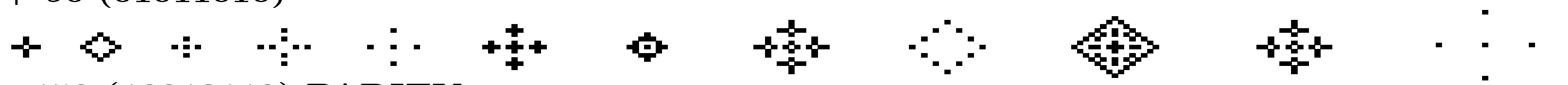
+ 54 (00110110)



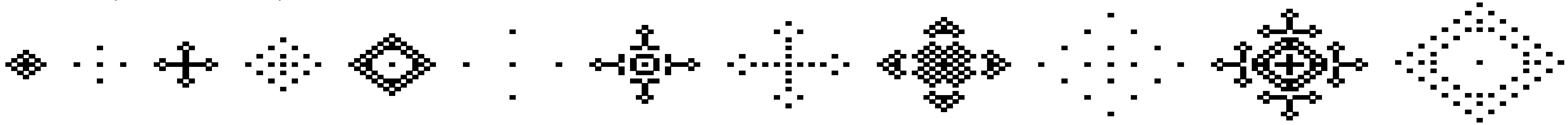
+ 76 (01001100)



+ 90 (01011010)



+150 (10010110) PARITY

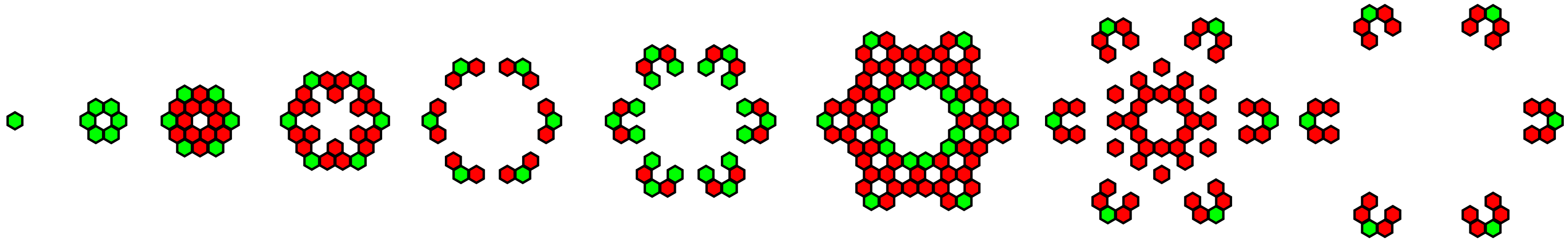


THREE STATES $\{0, 1, 2\}$: [**10**]

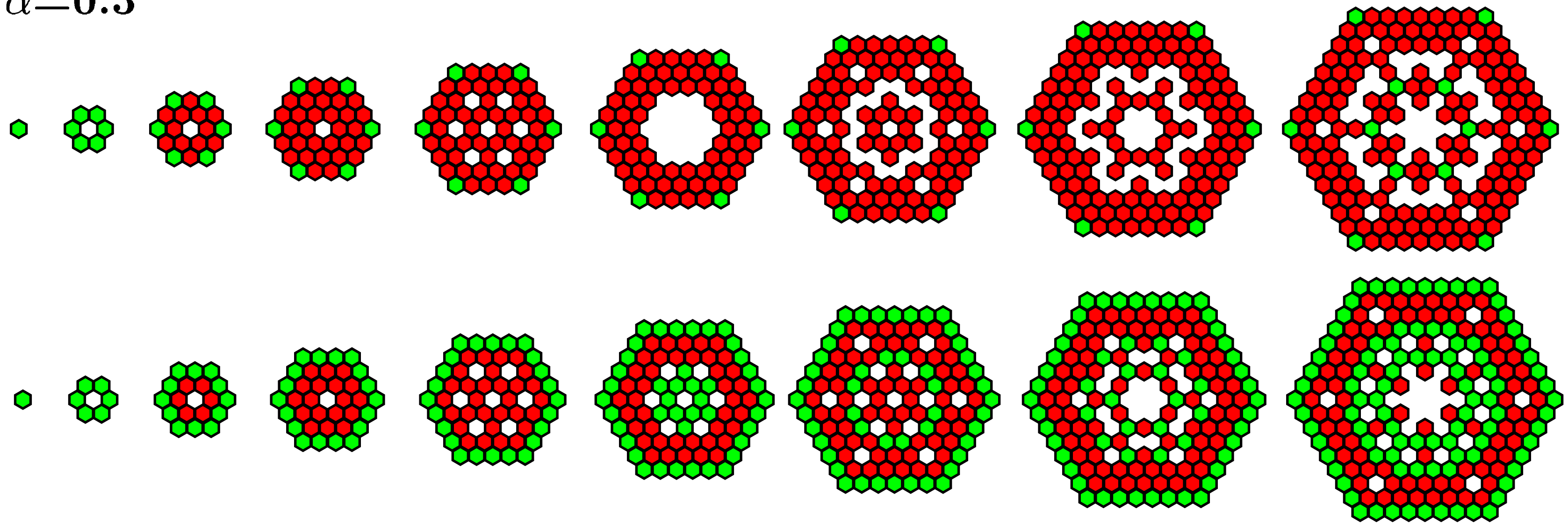
$$s_i^{(T)} = \begin{cases} 0 & if \;\; m_i^{(T)} < 0.5 \\ \sigma_i^{(T)} & if \;\; m_i^{(T)} = 0.5 \\ 1 & if \;\; 0.5 < m_i^{(T)} < 1.5 \\ \sigma_i^{(T)} & if \;\; m_i^{(T)} = 1.5 \\ 2 & if \;\; m_i^{(T)} > 1.5 \end{cases}$$

$k = 3$: α**-MEMORY EFFECTIVE if** $\alpha > 0.25 = \frac{1}{2(k-1)}$

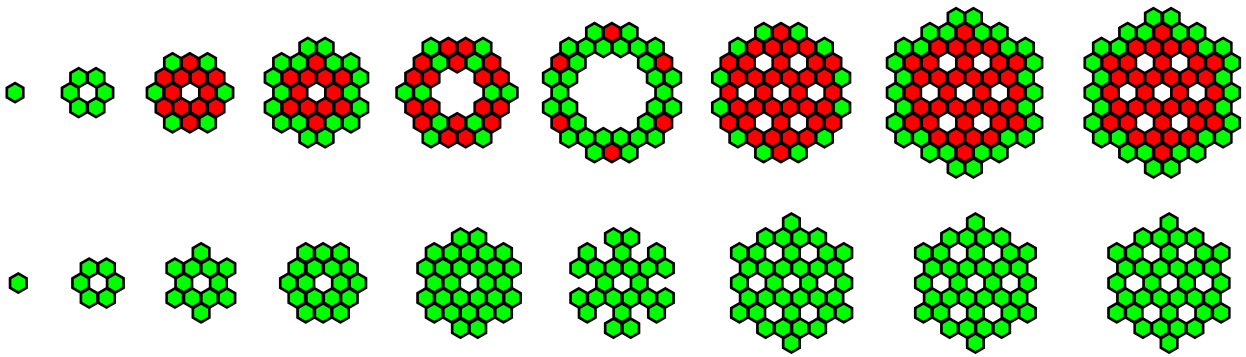
Ex.- The BEEHIVE Rule [4]



α=0.3



α=1.0



REVERSIBLE CA with MEMORY [12]:

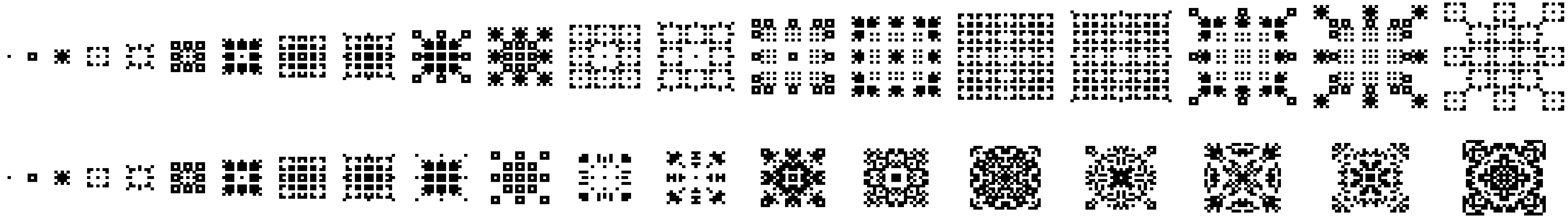
$$\sigma_i^{(T+1)} = \phi\big(s_j^{(T)} \in \mathcal{N}_i\big) \ominus \sigma_i^{(T-1)}$$

$$s_i^{(T-1)} = round\Big(\frac{\omega_i^{(T-1)} = (\omega_i^{(T)} - \sigma_i^{(T)})/\alpha}{\Omega(T-1)}\Big)$$

$$\sigma_i^{(T-1)} = \phi\big(s_j^{(T)} \in \mathcal{N}_i\big) \ominus \sigma_i^{(T+1)}$$

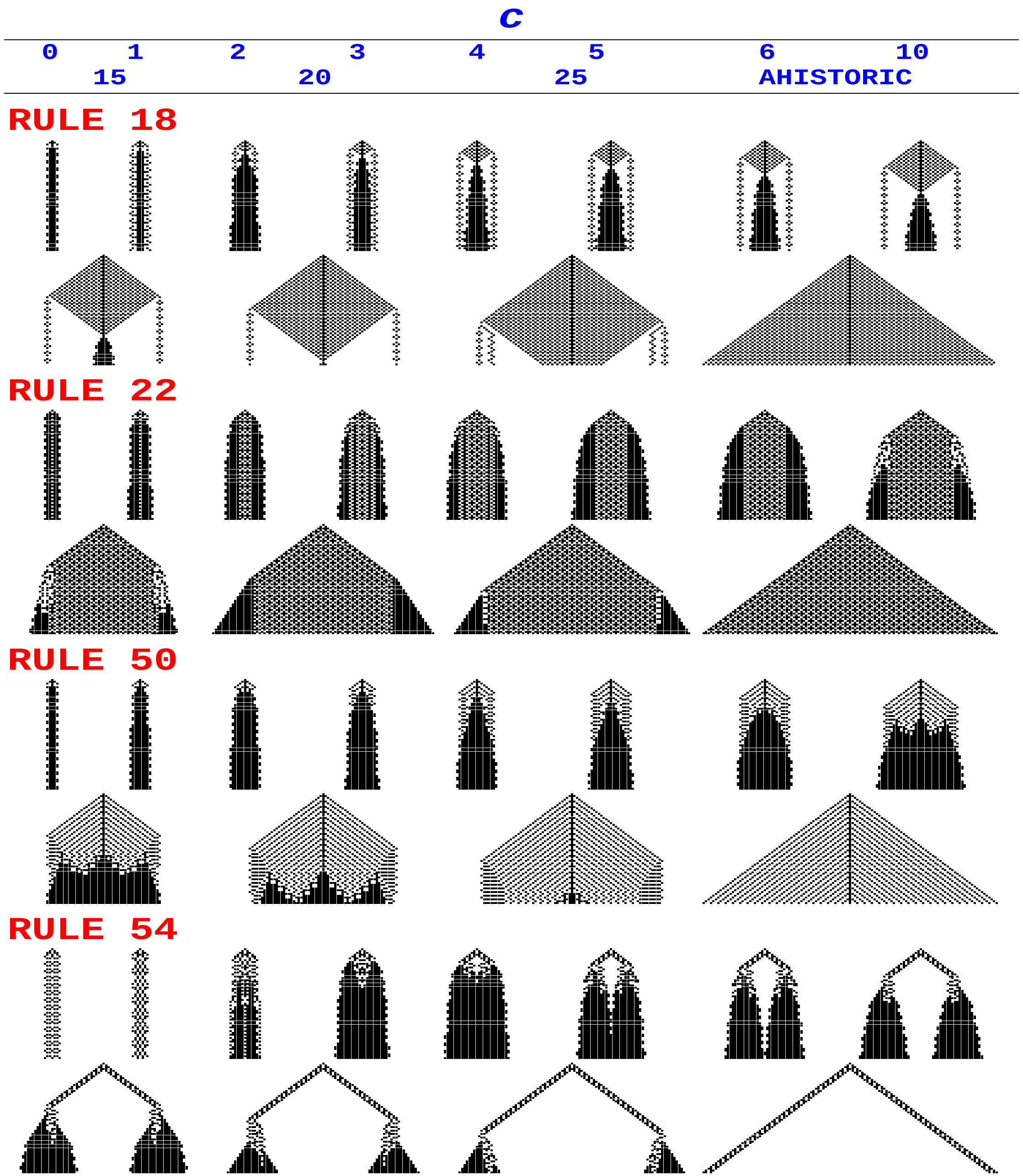
Reversible Parity Rule $(\{\sigma^{(0)}\} = \{\sigma^{(1)}\})$

NO MEMORY



α**=0.501**

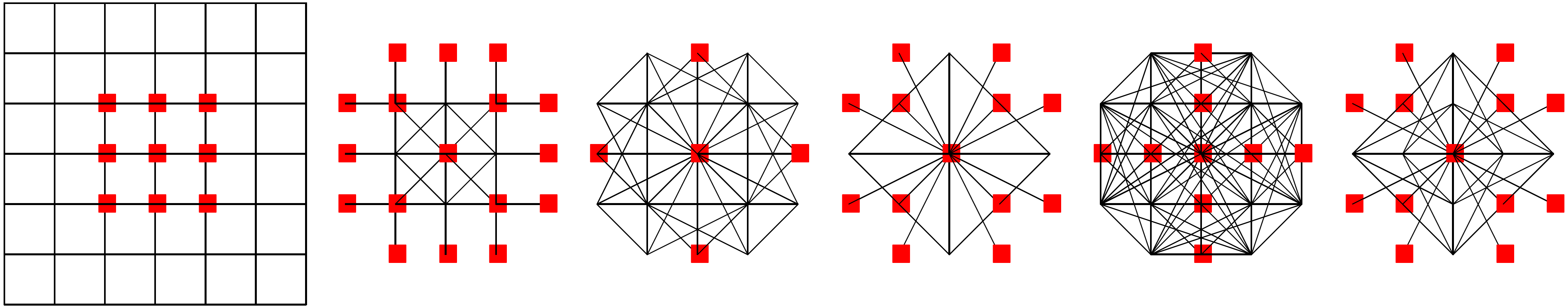
Reversible ER with $\delta = t^c$ memory



STRUCTURALLY DYNAMIC CA (SDCA)

State and link config. are both dynamic, altering each other

Example



Mass Parity rule (mod 2)

Links

Coupling

Add links between next-NN sites in which both values are 1

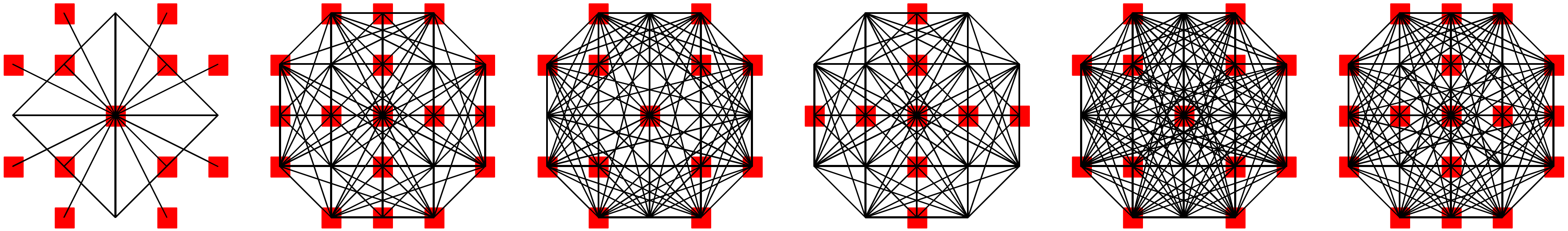
Decoupling

Remove links connected to sites in which both values are 0

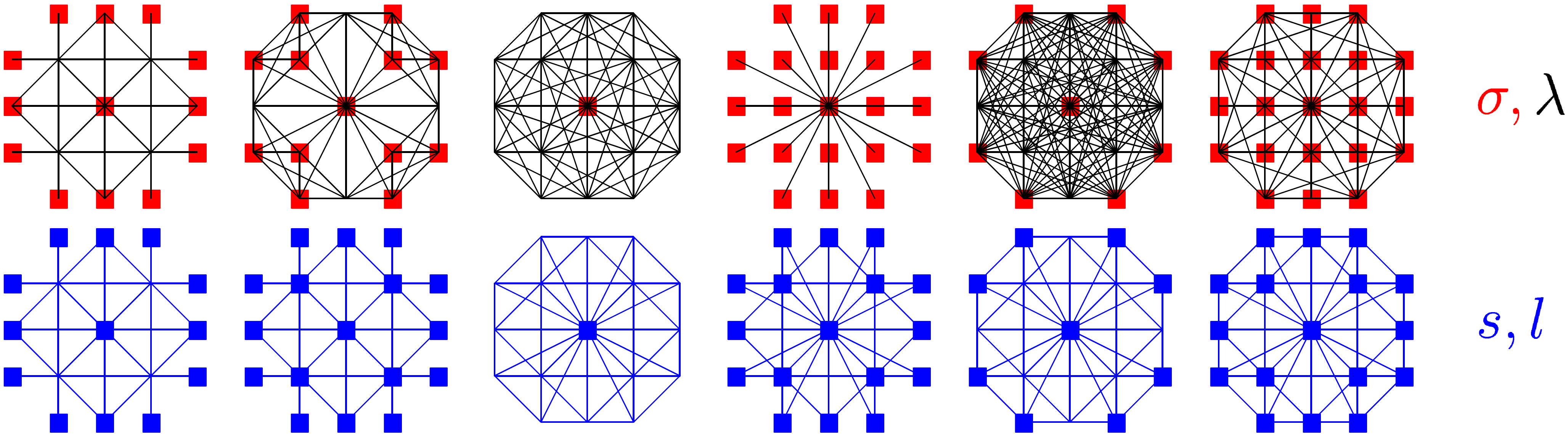
SDCA with MEMORY [3]

$$\sigma_i^{(T+1)} = \phi(s_j^{(T)} \in N_i^{(T)}) \qquad \lambda_{i,j}^{(T+1)} = \psi(s_i^{(T)}, s_j^{(T)}, \{l^{(T)}\})$$

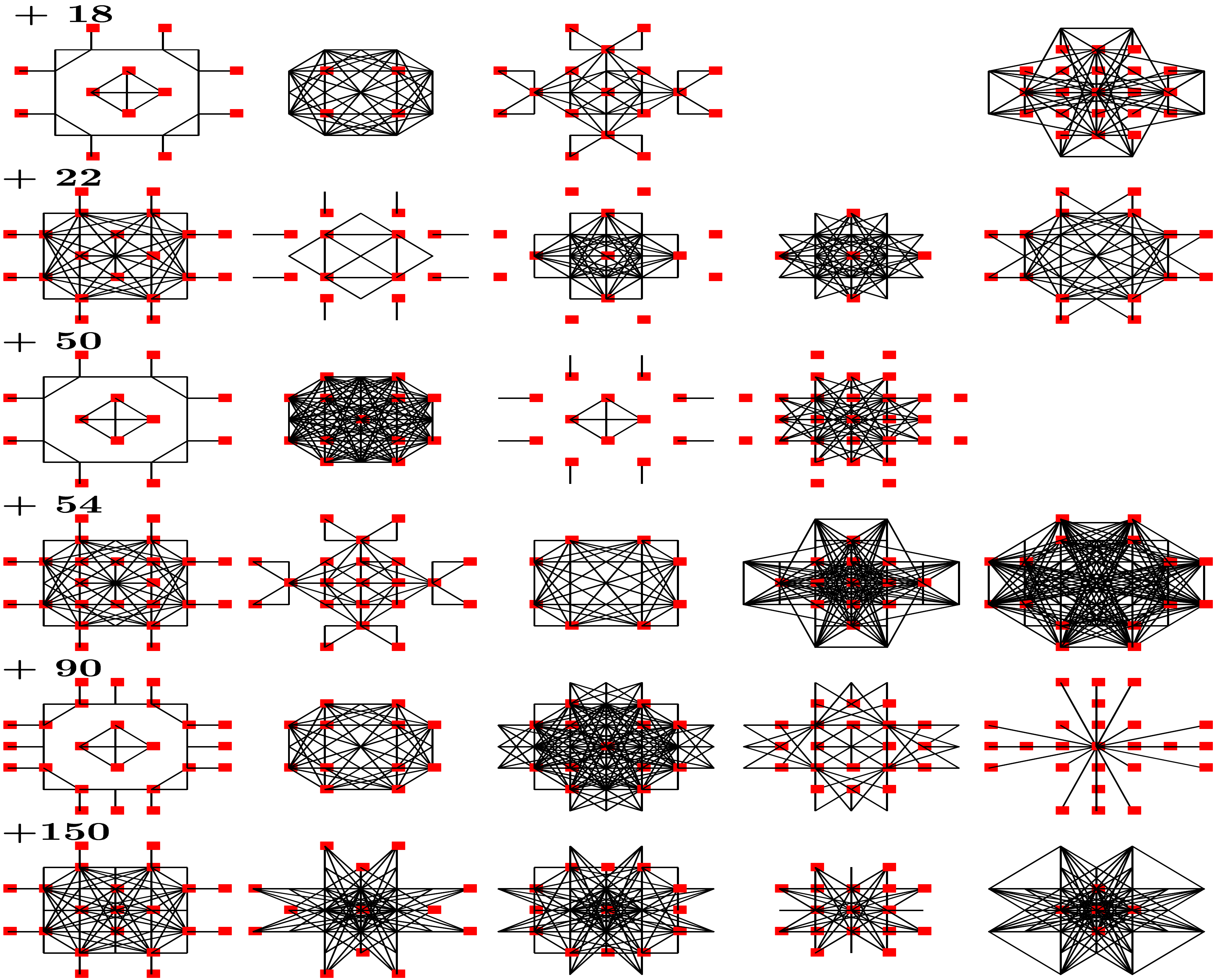
$\alpha = 0.6 \qquad T = 3 - 8$



$\alpha = 1.0 \qquad T = 3 - 8$



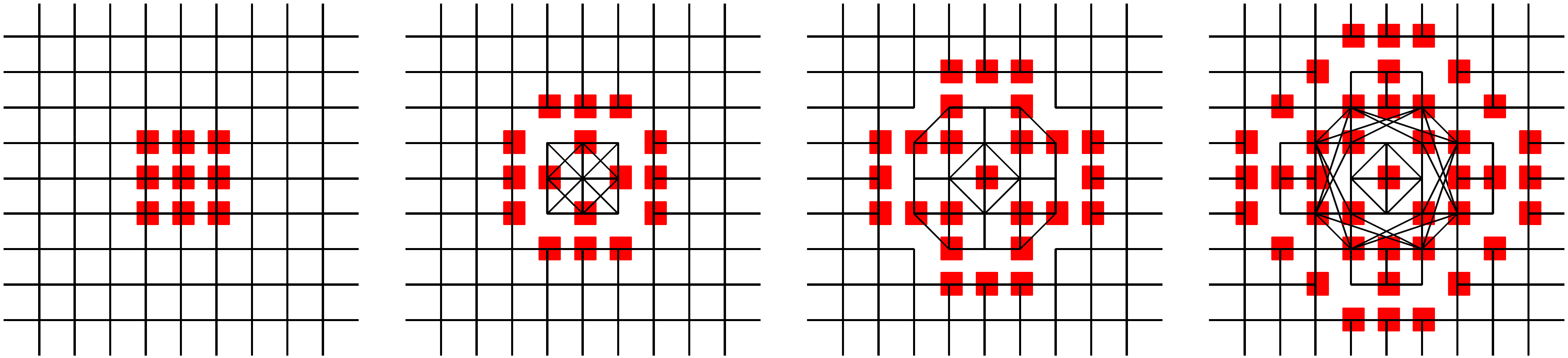
The SDCA Parity rule with Elementary Rules as Memory



REVERSIBLE SDCA [2]

$$\sigma_i^{(T+1)} = \phi\big(\sigma_j^{(T)} \in \mathcal{N}_i^{(T)}\big) \ominus \sigma_i^{(T-1)}$$

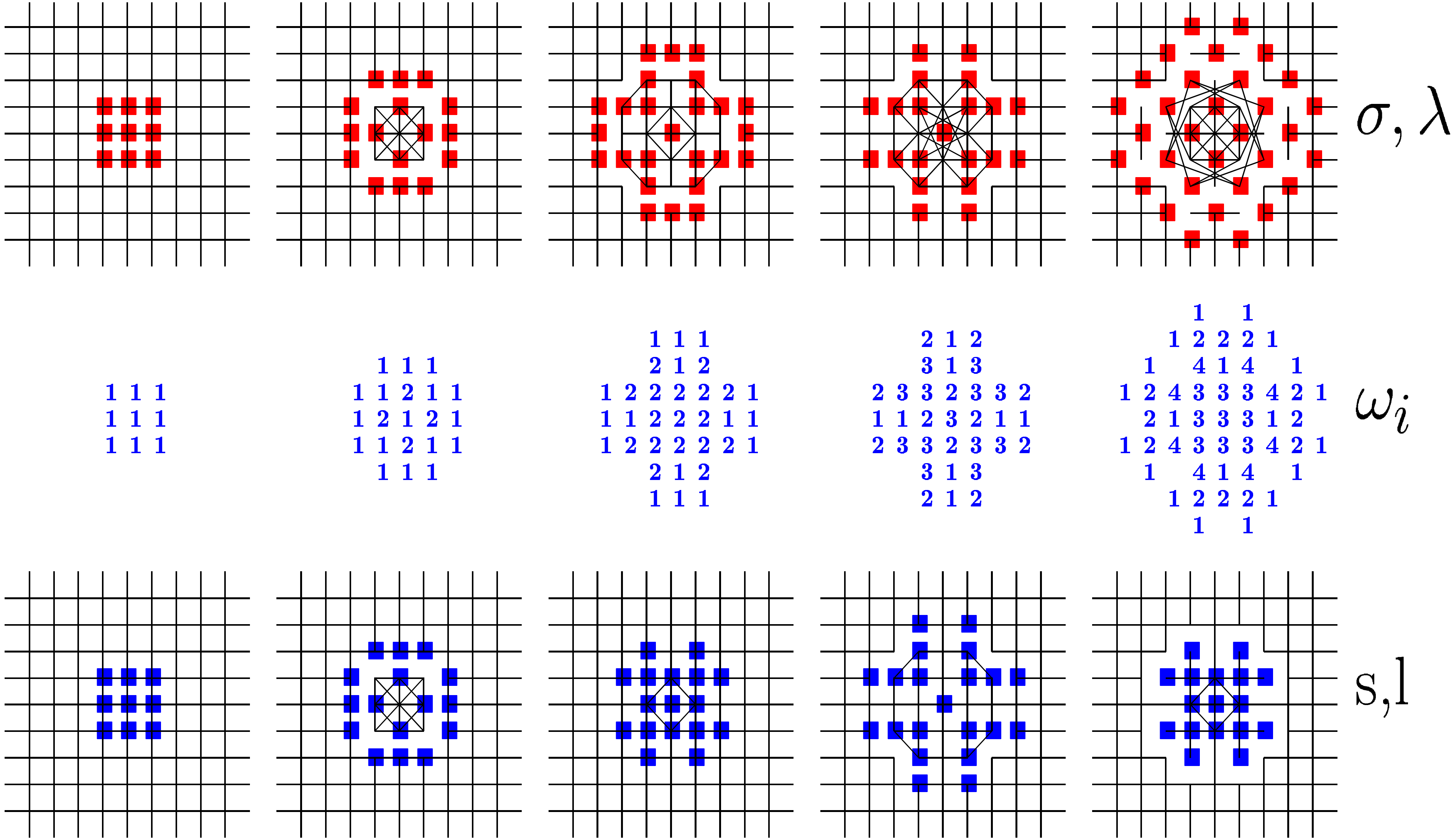
$$\lambda_{i,j}^{(T+1)} = \psi\big(\sigma_i^{(T)}, \sigma_j^{(T)}, \{\lambda^{(T)}\}\big) \ominus \lambda_{i,j}^{(T-1)}$$



REVERSIBLE SDCA with MEMORY

$$\sigma_i^{(T+1)} = \phi\left(s_j^{(T)} \in N_i^{(T)}\right) \ominus \sigma_i^{(T-1)} \quad , \quad \lambda_{i,j}^{(T+1)} = \psi\left(s_i^{(T)}, s_j^{(T)}, \{l^{(T)}\}\right) \ominus \lambda_{i,j}^{(T-1)}$$

FULL MEMORY ($\alpha = 1.0$)



There is plenty of room *with simple memory* :
Unaltered transition rule(function of previous states)

- **Probabilistic CA** [6]**:** $p = P\left(\sigma_i^{(T+1)} = 1 \Big/ s_{i-1}^{(T)}, s_i^{(T)}, s_{i+1}^{(T)}\right)$

- **Heterogeneous CA (BN) :** $\sigma_i^{(T+1)} = \phi_i\left(s_j^{(T)} \in \mathcal{N}_i\right)$

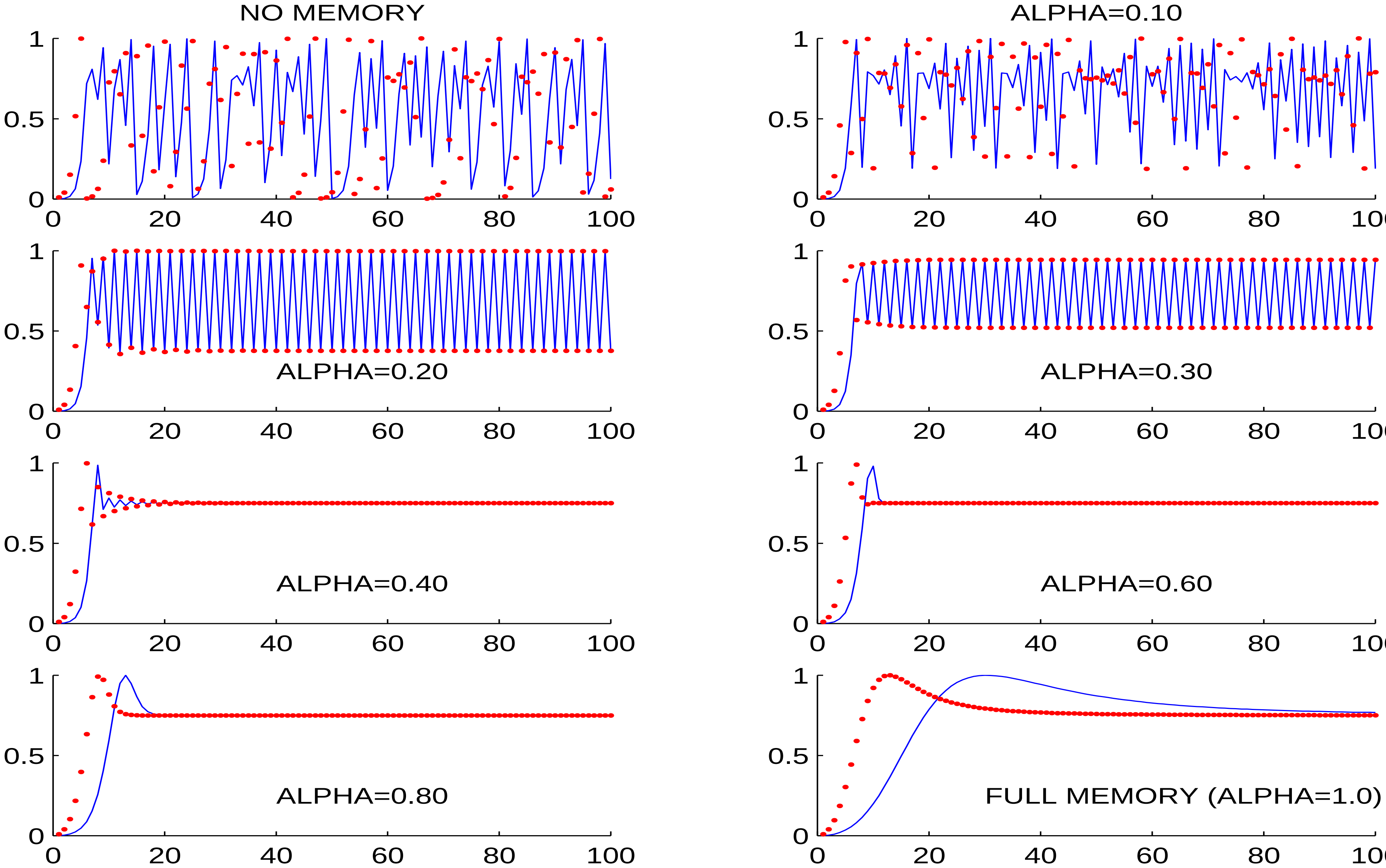
- **Continuous CA (CML) :** $\sigma_i^{(T+1)} = \varphi\left(m_j^{(T)} \in \mathcal{N}_i^{(T)}\right)$

- **Discrete Dynamical Systems :** $x_{T+1} = f(m_T)$

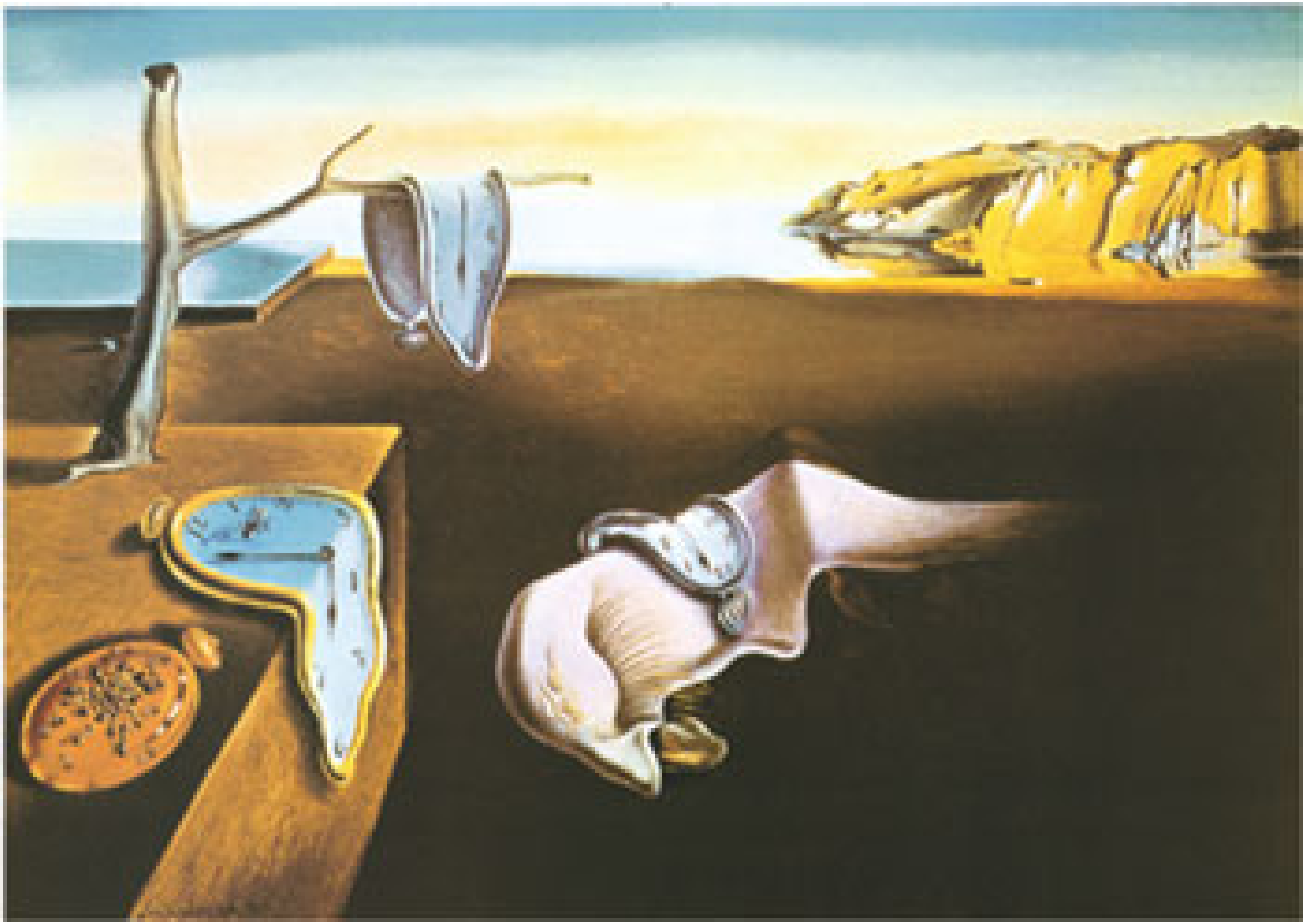
$$m_T = \frac{x_T + \sum\limits_{t=1}^{T-1} \alpha^{T-t} x_t}{1 + \sum\limits_{t=1}^{T-1} \alpha^{T-t}} \equiv \frac{\omega_T}{\Omega(T)}$$

The LOGISTIC map with memory [10]

$$x_{T+1} = 4m_T(1-m_T) \quad Fixed\ point: x = 0.75$$



SALVADOR DALI



The Persistence of Memory

Disintegration of the Persistence of Memory

References

[1] R.Alonso-Sanz,R.,Martin,M.(2006). A Structurally Dynamic Cellular Automaton with Memory in the Hexagonal Tessellation. El Yacoubi,S.,Chopard.B. (Eds.): ACRI-2006, LNCS (in press).

[2] Alonso-Sanz,R.(2006). Reversible Structurally Dynamic Cellular Automata with Memory: a simple example. J. of Cellular Automata (in press).

[3] Alonso-Sanz,R.(2006). A Structurally Dynamic Cellular Automaton with Memory. *Chaos, Solitons & Fractals* (in press).

[4] Alonso-Sanz,R.(2006). The Beehive Cellular Automaton with Memory. *J. of Cellular Automata* (in press).

[5] Alonso-Sanz,R.,M.Martin. (2006). Elementary Cellular Automata with Elementary Memory Rules in Cells: the case of linear rules. *J. of Cellular Automata*, 1, 70-86.

[6] R.Alonso-Sanz. Phase transitions in an elementary probabilistic cellular automaton with memory, *Physica A*, 347 (2005) 383-401.

[7] R.Alonso-Sanz,R.,Martin,M.(2004). Elementary Probabilistic Cellular Automata with Memory in Cells. P.M.A. Sloot et al. (Eds.): ACRI-2004, LNCS 3305, 11-20.

[8] R.Alonso-Sanz,M.Martin. One-dimensional Cellular Automata with Memory in Cells of the Most Frequent Recent Value , *Complex Systems*, (2005), 15, 203-236.

[9] R.Alonso-Sanz. One-dimensional, $r = 2$ cellular automata with memory, *Int.J. Bifurcation and Chaos*, 14(2004) 3217-3248.

[10] R.Alonso-Sanz,M.Martin. Three-state one-dimensional cellular automata with memory, *Chaos, Solitons and Fractals*, 21 (2004) 809-834.

[11] J.R.Sanchez,R.Alonso-Sanz. Multifractal Properties of R90 Cellular Automaton with Memory, *Int.J. Modern Physics C* 15 (2004) 1461.

[12] R.Alonso-Sanz. Reversible Cellular Automata with Memory. *Physica D*, 175 (2003) 1-30.

[13] R.Alonso-Sanz,M.Martin. Elementary cellular automata with memory, *Complex Systems*, 14 (2003) 99-126 .

[14] R.Alonso-Sanz,M.Martin. Cellular automata with accumulative memory: legal rules starting from a single site seed, *Int.J. Modern Physics* C, 14 (2003) 695-719.

[15] R.Alonso-Sanz,M.Martin. One-dimensional cellular automata with memory: patterns starting with a single site seed, *Int.J. Bifurcation and Chaos*, 12 (2002) 205-226.

[16] Alonso-Sanz,R.,Martin,M.(2003c). Cellular automata with memory. In Modeling of Complex Systems. Seventh Granada Lectures P.L.Garrido,J.Marrro (eds.). AIP Conf. Proc. 661,153-158.

[17] R.Alonso-Sanz,M.Martin. Two-dimensional cellular automata with memory: patterns starting with a single site seed, *Int. J. Modern Physics C*, 13 (2002) 49-65.

[18] Alonso-Sanz,R.,Martin,M.C.,Martin,M.(2001b). Historic Life. *Int. J. Bifurcation and Chaos*, 11,6,1665-1682.

[19] Alonso-Sanz,R.(2005). The Paulov versus Anti-Paulov contest with memory. *Int.J. Bifurcation and Chaos*, 15, 10, 3395-3407.

[20] Alonso-Sanz,R.,Martin,M.(2006). Memory Boosts Cooperation. *Int.J. Modern Physics C*, 17,6,841-852.

[21] Alonso-Sanz,R.,Martin,M.(2005). The spatialized Prisoners Dilemma with limited trailing memory . In Modeling Cooperative Behaviour In the Social Sciences: Eigth Granada Lectures. P.L.Garrido et al. (eds.). AIP Conf. Proc. 779, 124-127.

[22] Alonso-Sanz,R.,Martin,M.C.,Martin,M.(2001d). The Effect of Memory in the Spatial Continuous-valued Prisoner's Dilemma. *Int. J. Bifurcation and Chaos*, 11,8,2061-2083.

[23] Alonso-Sanz,R.,Martin,M.C.,Martin,M.(2001c).The Historic-Stochastic Strategist. *Int. J. Bifurcation and Chaos*, 11,7,2037-2050.

[24] Alonso-Sanz,R.,Martin,M.C.,Martin,M.(2001a). The Historic Strategist. *Int. J. Bifurcation and Chaos*, 11,4,943-966.

[25] Alonso-Sanz,R.,Martin,M.C.,Martin,M.(2000). Discounting in the Historic Prisoner's Dilemma. *Int.J. Bifurcation and Chaos*, 10,1, 87-102.

[26] Alonso-Sanz,R.(1999b). The Historic Prisoner's Dilemma. *Int.J. Bifurcation and Chaos*, 9, 6, 1197-