About Coarse Graining by elimination of relevant degrees of freedom

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Abstract:

A different way to coarse grain elementary Cellular Automaton, based on application of renormalisation procedure and developped by Goldenfeld and Israeli, finish, independantly from authors, to be presented as saving us all from the grip of unpredictability--from the infamousrule 110 (http://cse.ucdavis.edu/~chaos/news/).

I want to analyse the originally paper and propose consequences more realistic.

Results:

Results of the conducted analyzes and experiments have shown why the AC emulation map of Goldenfeld and the AC emulation map of Wolfram are differents.

Conclusions:

The work carried out has demonstrated that the difference is not fundamental and the magnification of this difference by scientific magazine like NewScientist seems to come not from scientific reason, as pretended. This seems to be a sign that NKS start to diffuse out of the strict scientific field, and mobilize other fields of knowledge to answer it.

Introduction

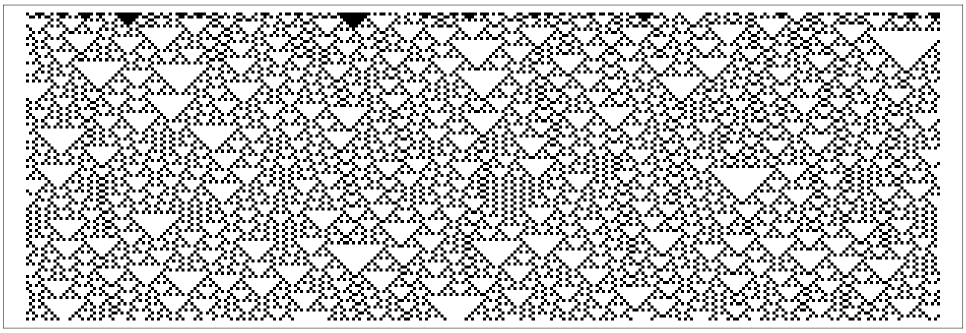
Aims: Check by myself the Goldenfeld's proposition that his method of coarse graining is able to transform an irreductible elementary Cellular Automaton (CA) in predictible CA, to swap from class 3 or 4 into class 1 or 2, if no precision is required.
see "Computational Irreducibility and the Predictability of Complex Physical Systems" Goldenfeld / Israeli PHYSICAL REVIEW LETTERS-2003

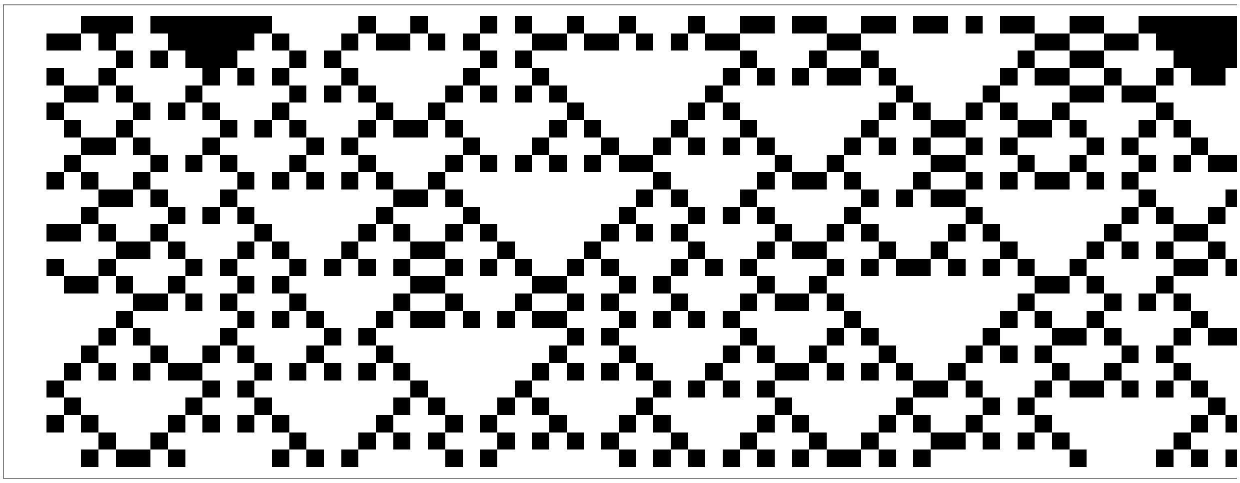
Two method for coarse graining CA

→ By elimination of relevant degrees of freedom (Goldenfeld-Israeli)

RULE 146 class 3

```
ArrayPlot[
 CellularAutomaton[146, Table[Random[Integer], {300}], 100]]
```

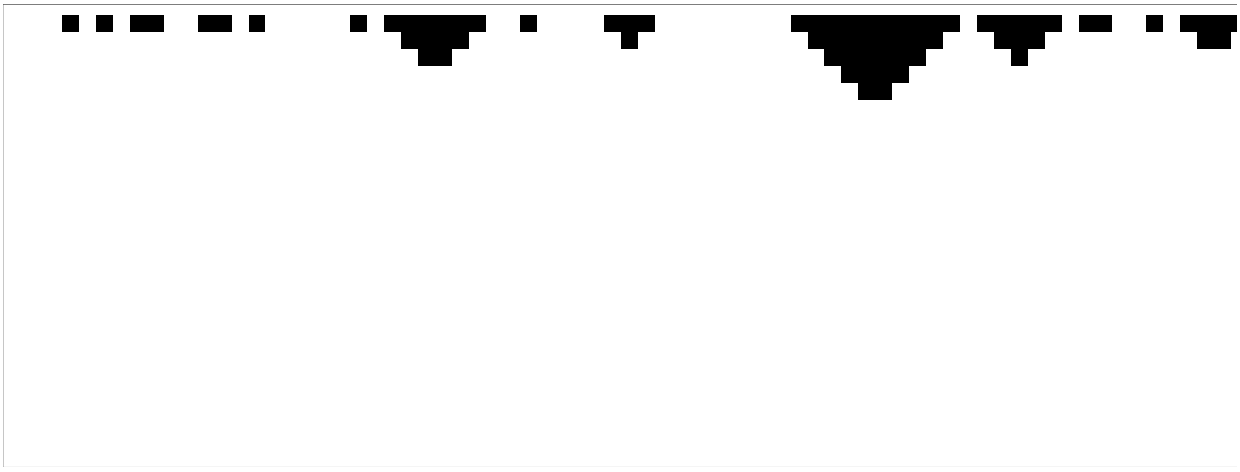




RULE 146 (detail)

RULE 146 coarse-grained OR RULE 128 class 1

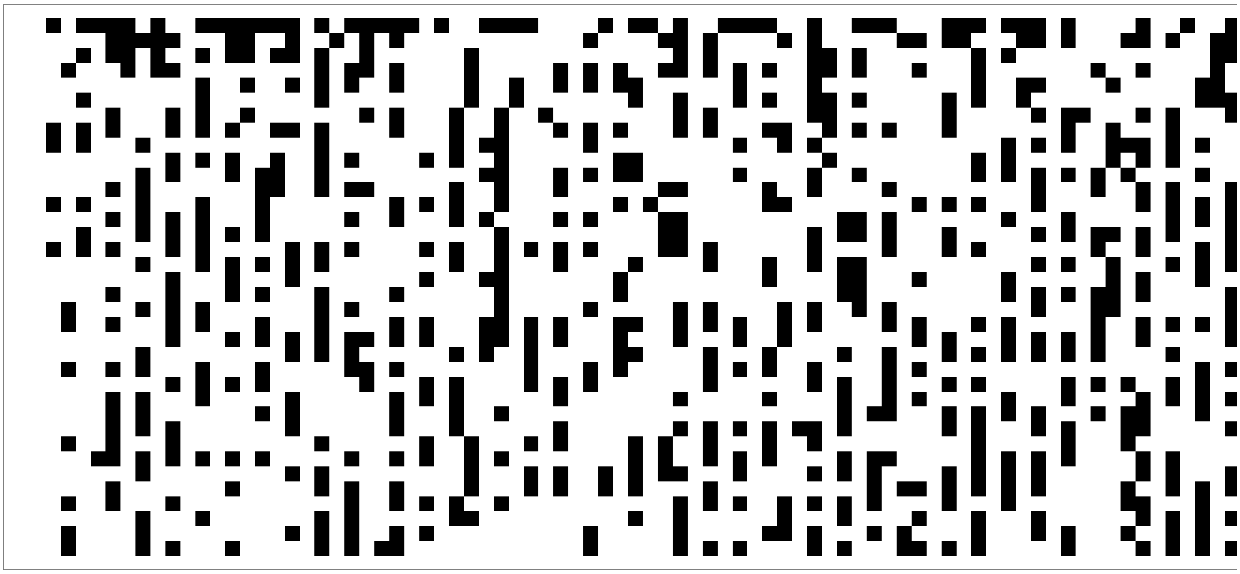
```
ArrayPlot[
 CellularAutomaton[128, Table[Random[Integer], {100}], 25]]
```



→ By elimination of irrelevant degrees of freedom (NKS)

RULE 146 coarse-grained by this way = still Class 3

```
ArrayPlot[
 Take[CellularAutomaton[146, RandomInteger[{0, 1}, 800], 500],
  {1, -1, 14}, {1, -1, 7}]]
```



What is coarse graining, in those both cases?

. Reduction using supercells

. Block of cells of original CA are representing by only one cell in coarse graining CA

What is coarse graining, in general?

. Do transformations to extract some characteristic
. Any idea of transformation is worth to test

Renormalisation?

A trick to escape from difficult situations. When you have infinite to handle in physics e.g., we don’t know exactly why we can have some results doing some reduction, but everybody use it.
But sometimes, it seems to be like add more complexity to contourn complexity.

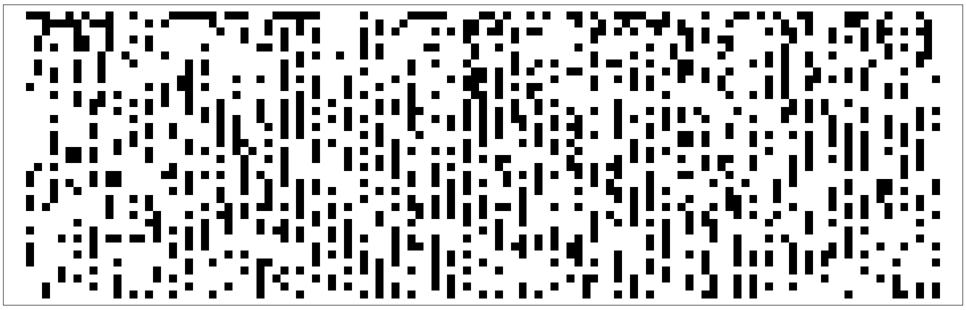
◻ Goldenfeld’s METHOD:

. Choose a size N for supercell (compact)
. Choose a rule P to simplify blocks into supercells (evolve)
. Test if the rule P give always the same average supercell for all combinations possible of 3N cells
. If ok, rule P and size N are accepted for coarse-graining method

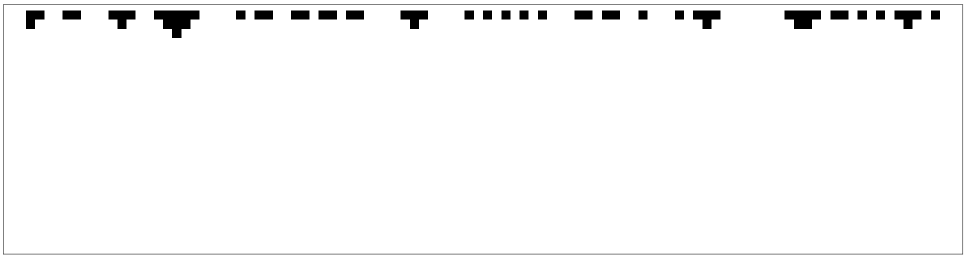
But it exists 2 ways to confirm the test:

compact, then evolve
or
evolve, then compact

Compact, then evolve (1) - NKS:
. only redundant information is lost
. this is the case of emulation blocking transformations, where no long time trajectories are lost



Evolve, than compact (2) - Goldenfeld:
. relevant information is lost
. Some information about long tim dynamics are lost



- A Logic formulation of coarse graining by eliminitation of irrelevant (Wolfram-1) and relevant (Goldenfeld-2) degrees of freedom.

$$\overline{A} = \overline{B} \qquad f(A) = f(B) \quad \xrightarrow{\text{yes}} \quad \overline{f(A)} = \overline{f(B)} \qquad (1)$$

$$\overline{A} = \overline{B} \qquad \overline{f(A)} = \overline{f(B)} \quad \xrightarrow{\text{no}} \quad f(A) = f(B) \qquad (2)$$

$\overline{x} \equiv P(x)$ is evolve $\quad f_x : \{S_x\}^3 \to \{S_x\}$ is compact

this is the reason why the two following maps are different

- Reactions, positions on forum NKS, about Goldenfeld/israeli's paper:

Jason Cawley's (Wolfram Research) experiment using applet of Israeli on rule 54
Jason told he coarse-grained rule 54 (with a result of 62 colors) and obtain a so complex pattern one could consider to use this rule like a good candidate to build a proof of universality for class3

Todd Rowland position (Wolfram Research)
Todd told roughly the opposite, noticing this method give so simple patterns, it is unuseful in order to understand anything about complex behavior

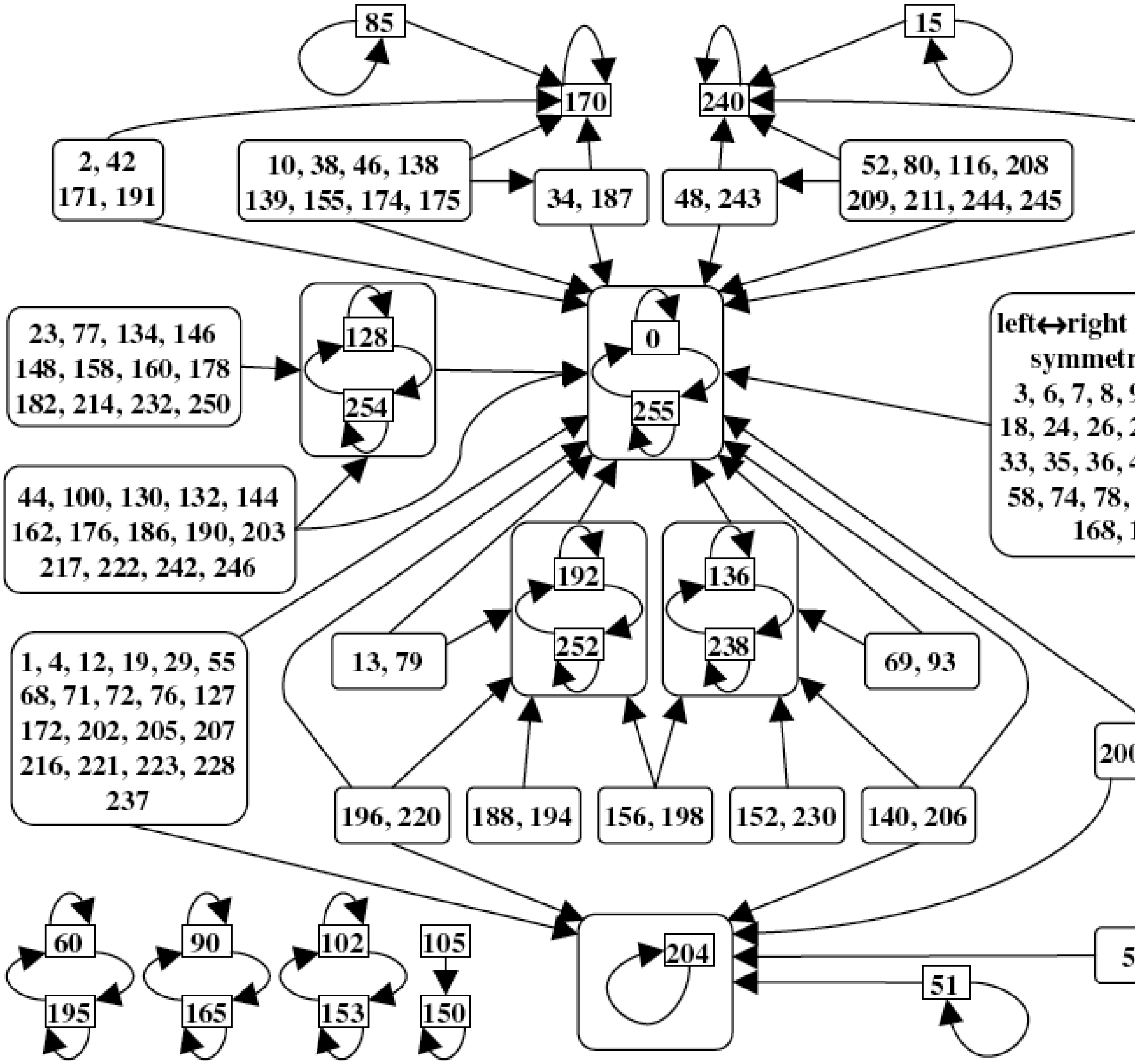
my opinion: Beetween those two apparently opposite positions, a large span of positions is possible. So let think that the story about this type of coarse graining is not finish, and by this way, the coarse graining in general remain an open problem.

Simulation of the Goldenfeld's method
see applet - java of Israeli
http://www.weizmann.ac.il/home/israeli/cgca.htms

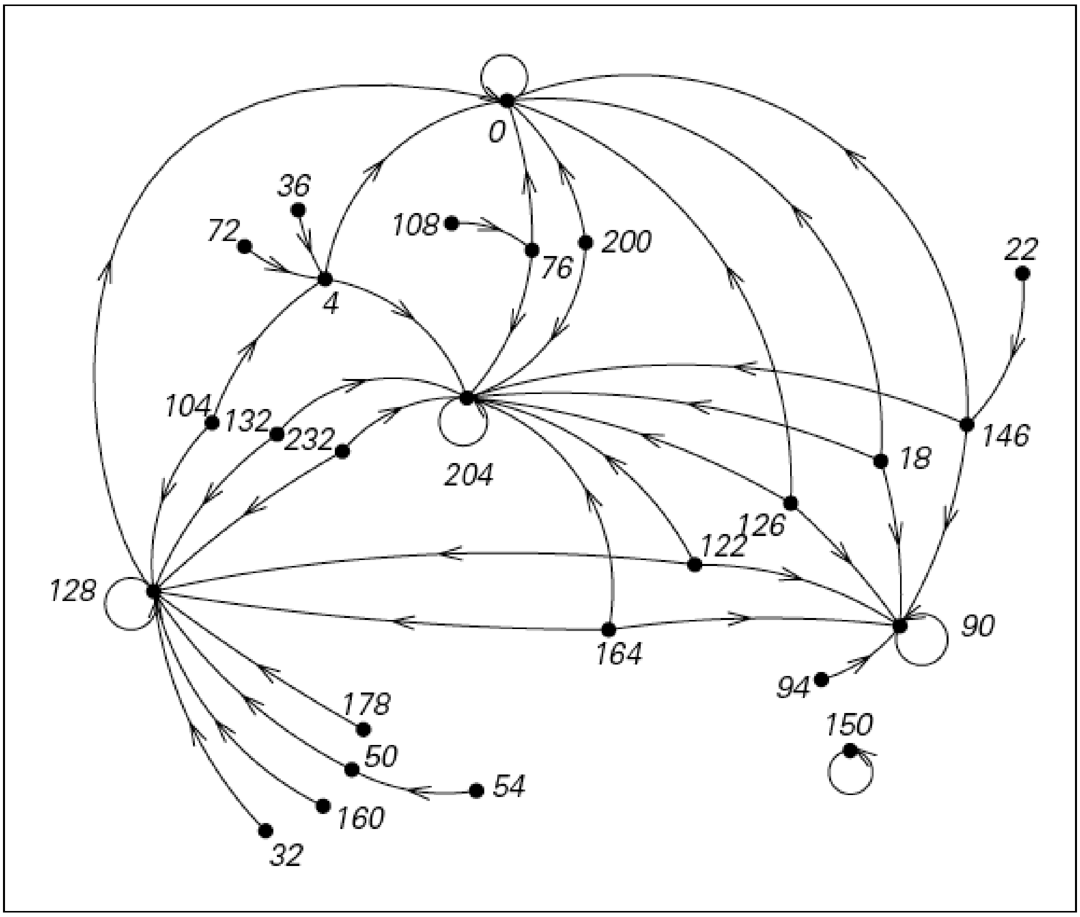
see Mathematica code written by Seth Chandler
http://forum.wolframscience.com/showthread.php?threadid=253

■ Goldenfeld / Wolfram maps emulations CA

Goldenfeld's map



Wolfram's map



Usefulness of Goldenfeld's methode - conclusion

Why not, in some cases? But it's not the revolution, and not something going to save us from the grip of impredicibility, as claimed by New-Scientist's paper.
In addition, results of this method, with greater N et P, are unpredictible and irreducible. We have to explore them to know them, to adopt an NKS methodologie, as exhaustive as possible.
Let explore the computationnal universe of coarse graining method, and all the other parts of the universe of computation, using method to direct access to irreductible systems like research of simples rules and eshaustive experiments on it.