Why numbers matter:
The construction of the number-line

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Beginning Assumptions

The proposal is that a 1-dimensional cellular automata can be used for modeling the correspondence between the real world and features of the arithmetic number line.

Numbers are a creation of our minds, while space- time has an independent reality that must be mapped out experimentally. The ontological framework used here is that all entities are parts or facets of one unified continuum

Energy/Matter

The postulate in this paper is that events and objects in the world are representing states of matter and energy. All matter and energy is a view of one whole. Energy is a human abstraction of states of objects or events that exist in the world. Matter and mass are assumed to be other states of the same entities that are measurable as states of energy.

Space-time is the arena on which all physical events take place. We observe that space-time displays discrete and continuous forms.

The real world in large scale and small scale does not agree with intuitive human ideas. Our model for real world events is not grounded in our human senses, but must be developed and tested by instruments of observation and measurement.

Number-line/Arithmetic

Simple rules lead to the number system. Computation proceeds by making changes in a coherent way. We create an individual number by selecting a portion of energy/matter we see as standing out of the energy/matter continuum. So a number is an abstract symbolic object we create based on real world events/objects.

We postulate each number being a discrete repeat of any other one. The labeling of a second "2" is an arbitrary human artifact. Some postulates about numbers: they are constructs of human thought; the number line as such is a model of representing and manipulating the symbolic number; some of the qualities that are expressed in number theory result from how we conceptualize numbers other than 1; and the elementary number theorems seem to be based in our experiences with the real world around us.

Numbers work because we can use language constructs to describe the world. We can record these constructs and take action based on them and cause a predicted effect in the real world. Objects, and events exist in the real world. Numbers can be described with language and used.

Cellular automata/Bridge Model

The postulated model here is that we can use cellular automata to model the correspondence between the real world and features of the arithmetic number line.

Cellular automata are discretely defined but exhibit continuous dynamics so are useful to address the discrete and continuous.

A cellular automata can model usable calculation algorithms. The elements for this bridging model include rules, procedures, transformation and rewriting. We need to model an arithmetic system with algorithmic methods. The algorithm contains numbers and follows a set of steps. Prediction of future states follows from a program's run. The program plus its initial condition moves to the end state.

The reason CA works as a bridge model is because:

A. it gives the correct results for basic arithmetic

B. it models actual real processes in the energy-matter continuum's
- i) algorithmic steps forming a sequence
- ii) individual units that change through the action of the steps

The change rules are modeling energy, space and time.

This model uses a 1 dimensional first order cellular automata with the following definition:

This 1D binary CA is defined as {Z,S,N,f,B} where

- Z can be finite or infinite
- $S = \{0,1\}$ is a set of two values
- $N = \{-1,0,1\}$ is the neighborhood of size $k = 3$ with symmetric radius $k0 = 1$
- f is a transition function rule
- $B = \{b-1, b1\}$ is the boundary

1 -> 000
2 -> 001
3 -> 010
4 -> 011
5 -> 100
6 -> 101
7 -> 110
8 -> 111

This CA is reversible.

This CA function mapping is bijective. And from H. Morita we know invertible Turing machines can be simulated by invertible cellular automata.

Conclusion

The conclusion is that the numbers, number-line and basic arithmetic are not Platonic objects but are human created tools.

It is shown that the usefulness of numbers can be explained by models of human language and models that explain the usefulness of the space-time postulate.

The number-line is stated to be a tool created by humans to be used in algorithms. Humans posit real world events that behave like numbers. Individual objects are posited to be useful within the present space-time theories.

A model that produces the traits of the real world, and of basic numbers and arithmetic is a simple universal calculating system like cellular automata. Cellular automata has traits that model the real world and can emulate and reproduce arithmetic methods.

The number-line models energy in real space with two restrictions:

1. the numbers are restricted to 2 directions
2. the cases are limited to full units (integers)

The number-line requires two aspects:

A. a symbol set, and
B. a program that uses the system set

A computable number is a number for which there is some program to compute it. Turing machines or cellular automatas can model such computation.

A model for 2 dimensional arithmetic can also be modeled with similar CA. Basic forms of behavior are modeled by cellular automata, Turing machines or lambda calculus. Arithmetic laws are validated in ways that other mathematical postulates are verified.

Future Steps

The linear number system may be a special case of a number system model that could be built that is 3 dimensional. This could lead to models that demonstrate some constants like pi and cosine and perhaps explain irrationals like square root(2).

Numbers arise because of the trait that processes have of being able to be sliced through a portion of time and change can be observed or likeness can be seen.

Numbers can be thought of as an abstract model that isolates a small part of the complete space/time of a process with measurable abstract objects.

There is perhaps work to be done using order theory to move from individual objects to the number-line.

We need a systematic approach to object identification. Such a system would be foundational to arithmetic. To have usable objects we must be able to individuate entities out from the continuity and to identify these individual entities over time

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