Code 52

Introduction

Code 52 shows a variety of behavior, some of which I'll show today. Starting with general behavior and then going to behavior in specified situations.

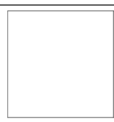
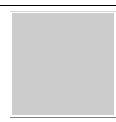
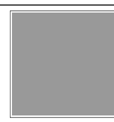
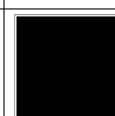
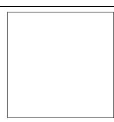
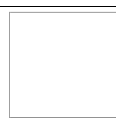
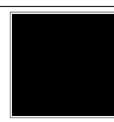
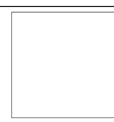
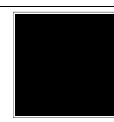
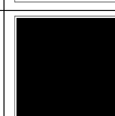
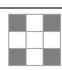
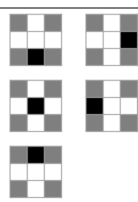
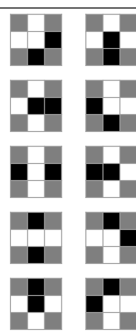
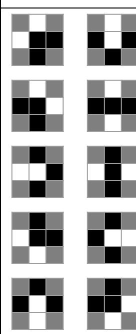
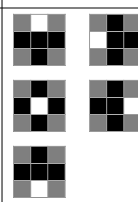
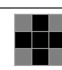
The main reason why I was attracted to code 52 was because it is relatively easy to find interesting behavior while also being relatively stable.

Part of the Principle of Computational Eqivalence tells us that complex behavior in a simple rule is likely to be universal. Code 52 has this sort of unpredictability. But even if we didn't have this principle, we might think that code 52 is universal since it shows great flexibility with finite computations, as well.

On the other hand, there is evidence that it may only be capable of finite computations. In the end though, I feel that it must be universal, that is, capable of emulating any computation indefinitely.

Rule description

The rule depends on teh total of the five neighbors, including the center cell. It is almost a moajority rule, except it differs when the average is closest to 2.5.

Total	0	1	2	3	4	5
Average						
Result						
Cases						

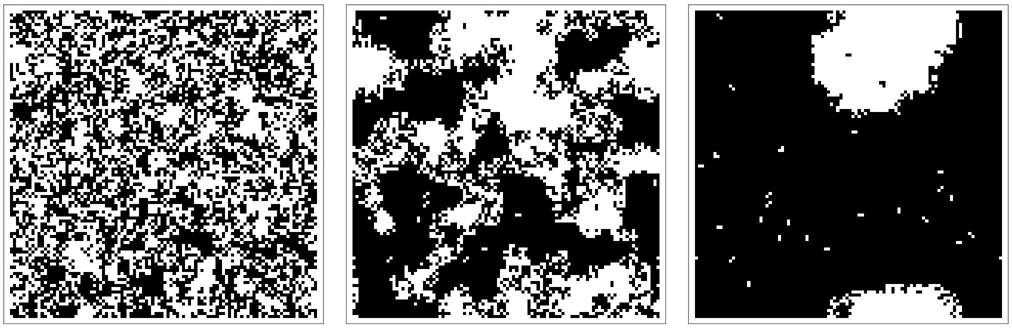
Past work

Code 52 gets mentioned in some of Stephen's early papers on CA's, and also in NKS.

For instance, he showed that a typical long term behavior involves a sort of clumping behavior [NKS, p.233] and that on a finite grid it can settle down into linear light and dark regions, with a boundary performing ECA rule 150.

Clumping behavior

10 steps | 100 steps | 1000 steps



random initials on size 100

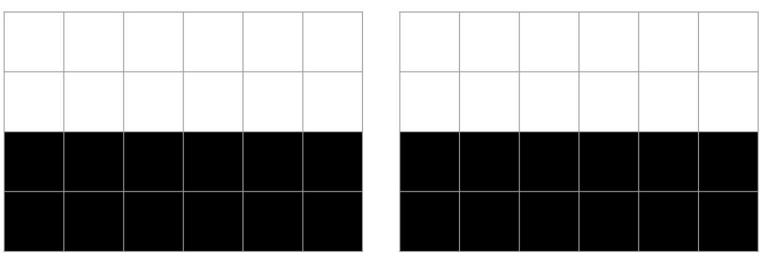
Also, Stephen has pointed out that the slices of code 52 obey a class 4 type of behavior. [NKS,p.692]

Trivial observations

The rule is symmetric with white and black.

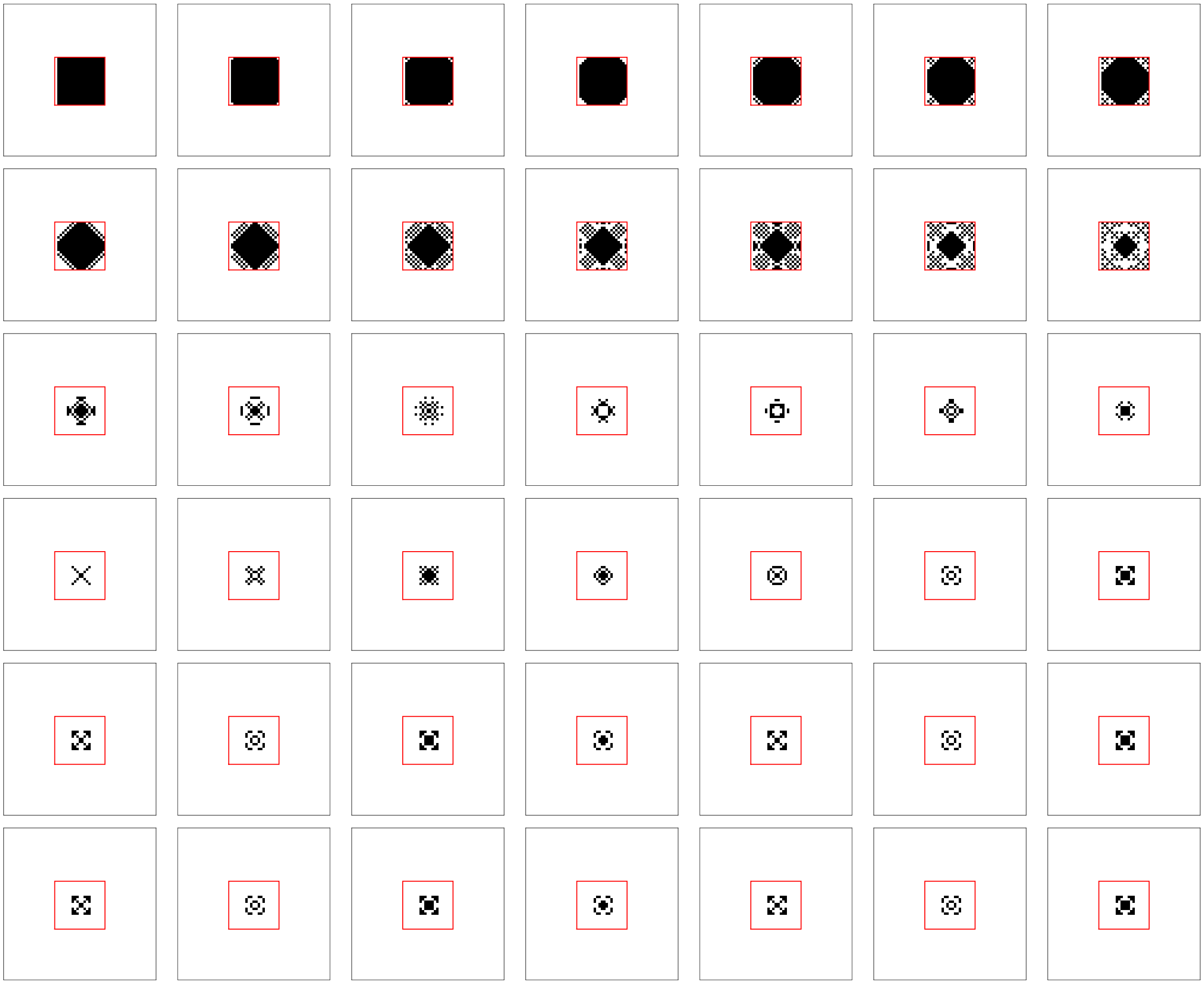
It is like a modified majority rule.

When there are walls between regions, thickness at least two, they are stable. Because at the boundary, within the black region there are always at least four black cells in each neighborhood.

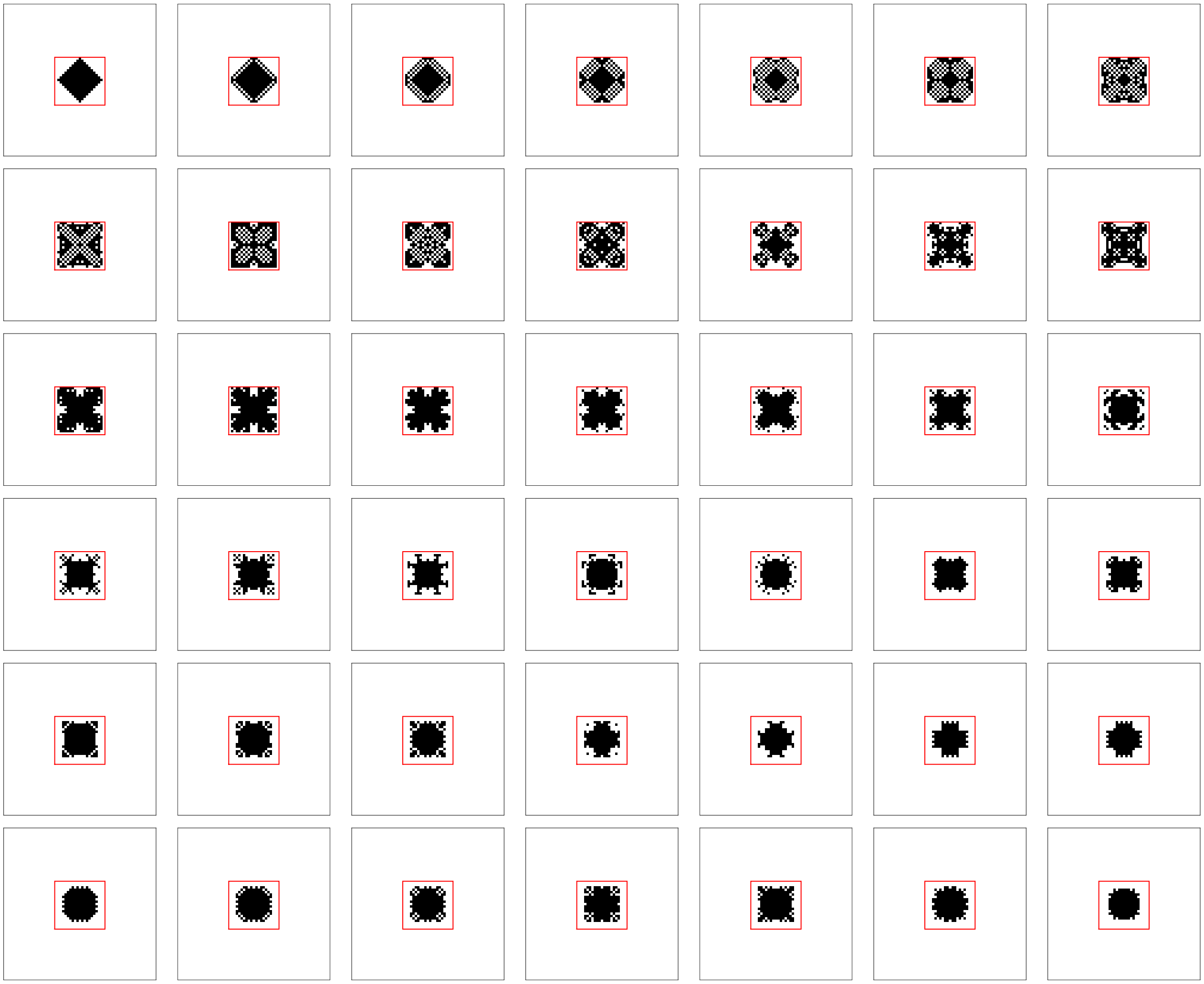


On a plain background there is no growth.

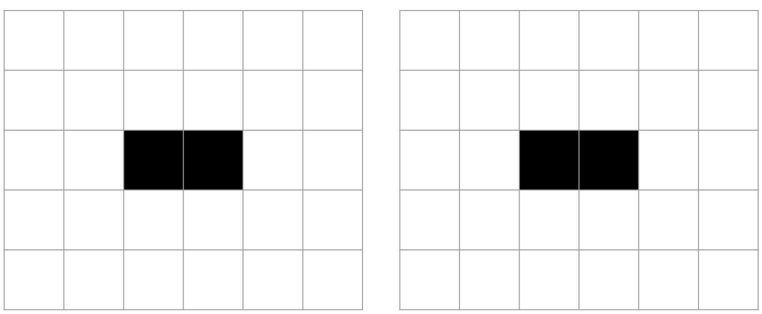
Any bounded region stays bounded. Here red marks the original boundary.



Another interesting case

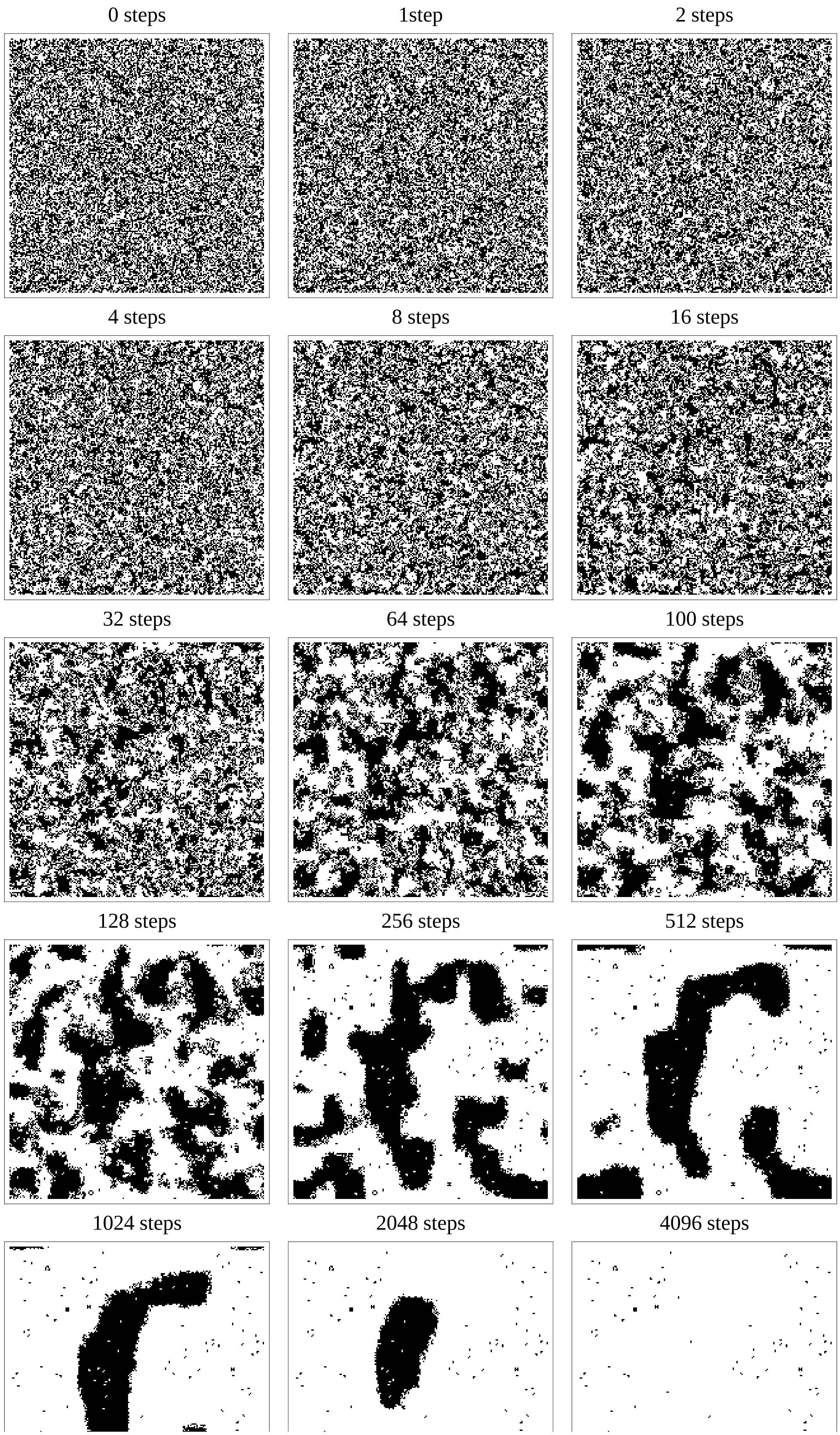


Stationary structures include two black cells.



Generic behavior

On a large grid with initial conditions 50% black or white, it shows a sort of clumping behavior, as mentioned previously.


0 steps
1step
2 steps
4 steps
8 steps
16 steps
32 steps
64 steps
100 steps
128 steps
256 steps
512 steps
1024 steps
2048 steps
4096 steps

first row on a size 200 grid for 400 steps

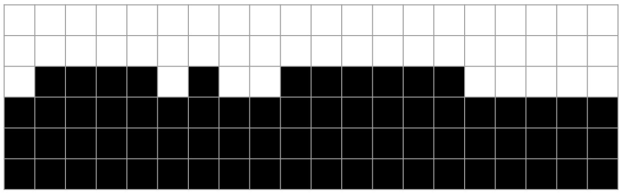
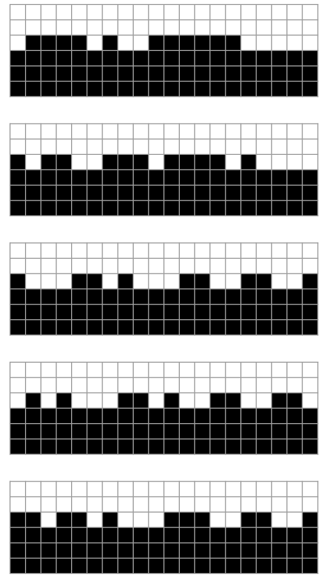
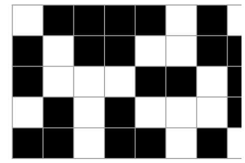
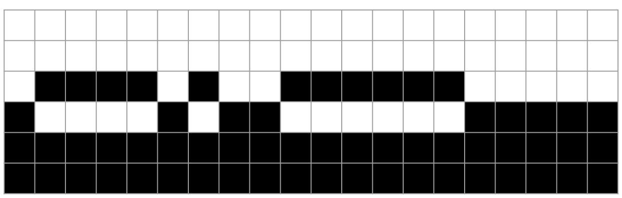
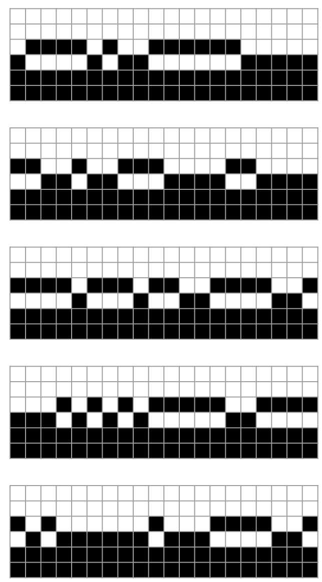
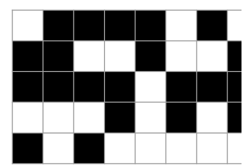
The testing was done on grids of size 40 by 40, with comparisons done on smaller grid sizes as well.

As time progress on a finite grid, it resolves, usually as either mostly white or black with a few stationary structures

About 25% of the time it resolves into a single band (at size 20). This is because it is on a finite grid.

The band persists, and most of the time has a boundary which follows rule 150, as noted in Stephen Wolfram [1985].

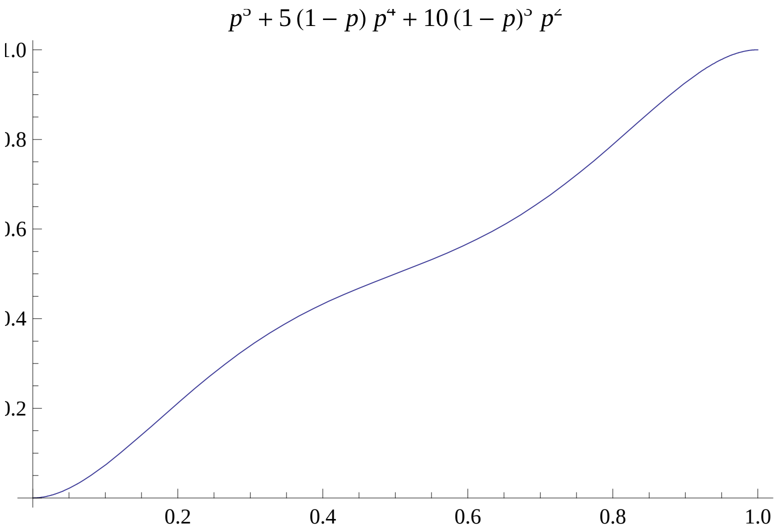
Somewhat more rare is this structure which follows rule 90, about 1/2 % of the time on grids of size 10 down to less than .01% on grids of size 20.

Emulates 150 		
Emulates 90 		

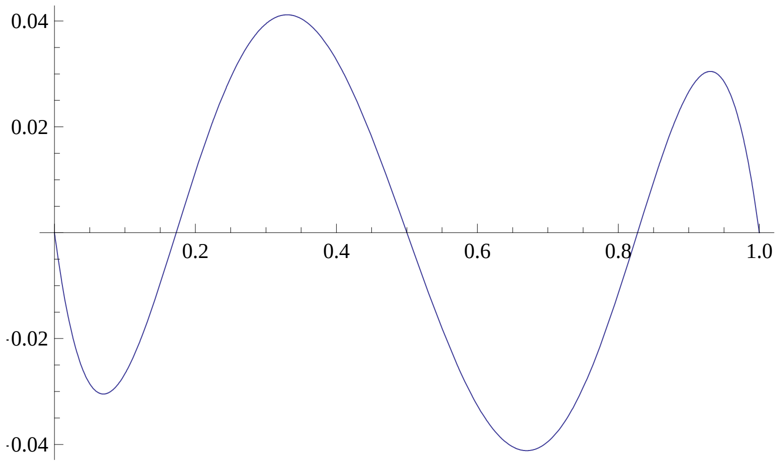
Theoretically, it is possible to have other boundaries, some following other ECA's, but I was not able to observe any coming from random initial conditions. I will come back to that later, when we get to the ECA emulations at the end of this talk.

Density behavior

In a mean-field approximation, one assumes the cell's colors are not correlated and are given by a uniform probability. The implications are that the density p updates by the following

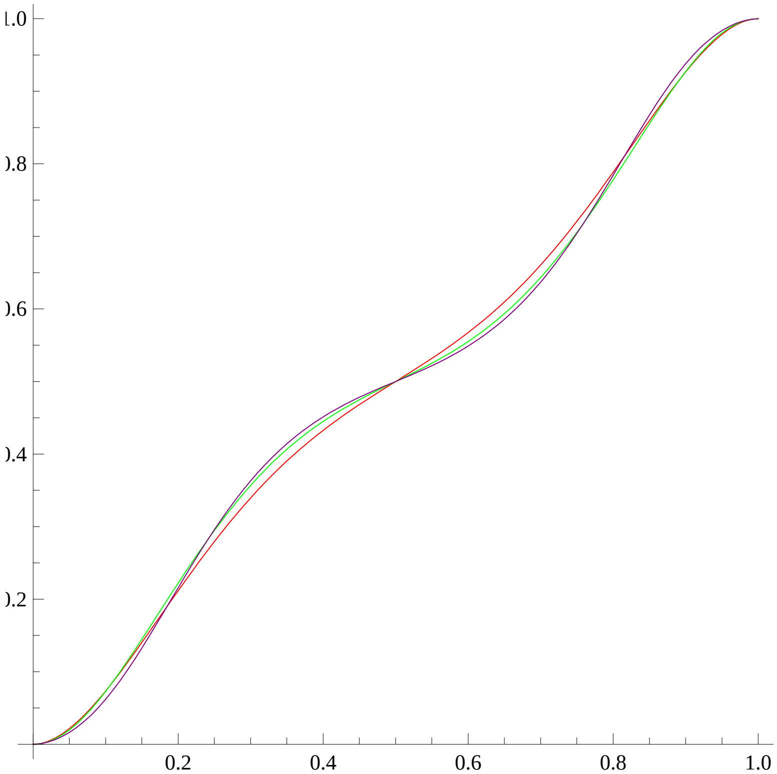


Uncorrelated density update after one step
likelihood of 5,4,or 2 neighbors being black



Difference from identity map

Looking more steps ahead



Red: Uncorrelated density update after one step

Green:Uncorrelated density update after two steps

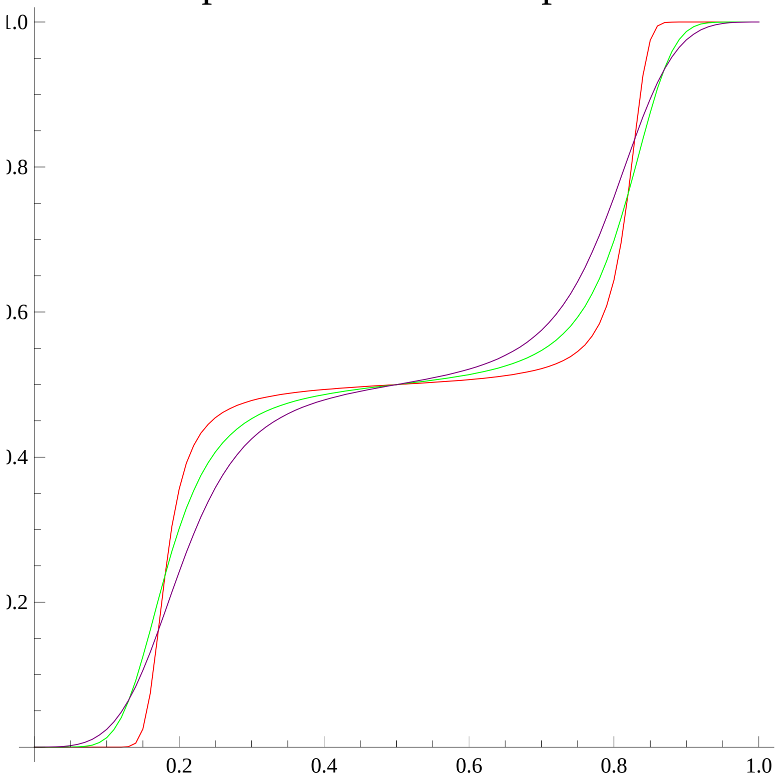
Purple: Uncorrelated density update after three steps

Recall that code 52 is symmetric with black and white, which explains the symmetry of the curve.

Note also that it is close to the straight line, and we can easily see its tendencies when repeated by subtracting that off. When it is less than about .17 it is decreasing, tending to zero, between .17 and .83 it tends to 1/2, and above that it tends to 1.

Iterating the map 6 times shows this prediction.

Three predictions for 6 steps ahead



Red: one step map (6x)

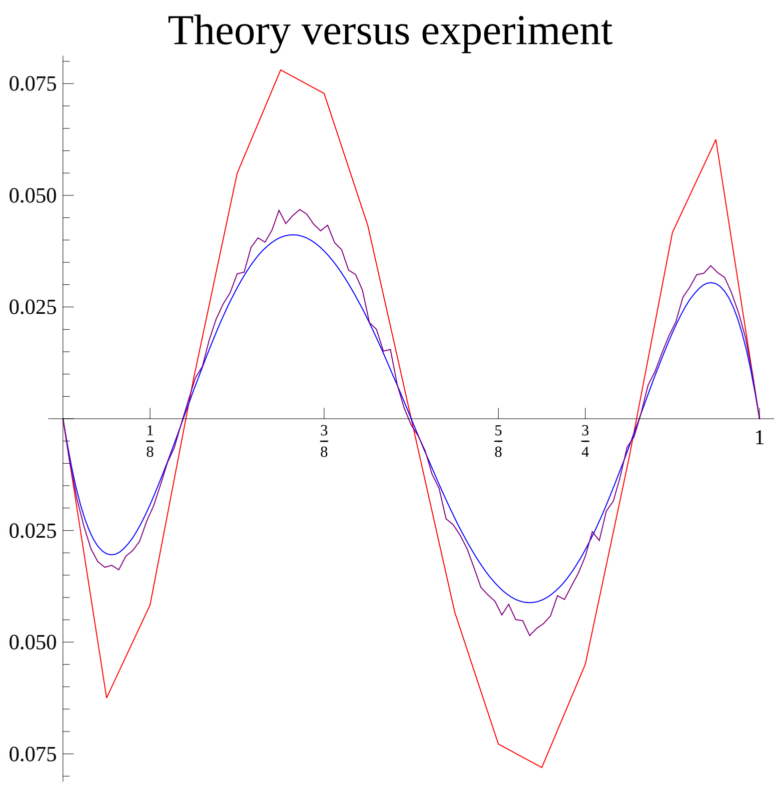
Green: two steps map (3x)

Purple: three steps map (2x)

As we can see it is pretty far from the experimental results.

Looking more steps ahead, in an attempt to make corrections as in NKS[p], shows that the experimental results are not a fluke.

We see that the mean field approximation is not right, which means that the rule is not as random as it appears. The arrangements are correlated.



Red: based on all 4x4 grids

Purple: random sample on 10x10 grid

Blue: theoretical prediction

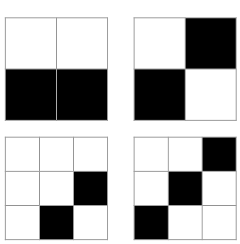
If one takes the main clumping behavior as an assumption, then it makes sense instead to assume only that the overall density is a value p, rather than the assumption that each cell is individually white or black with probability p. This gives a better analytical approximation to the actual behavior of the density function.

Other backgrounds

In examining this rule, we have just looked at the case of generic situations, but now let's look at specific configurations where code 52 does something special.

As in the study of the ECA, we can look for interesting behavior on cyclic backgrounds.

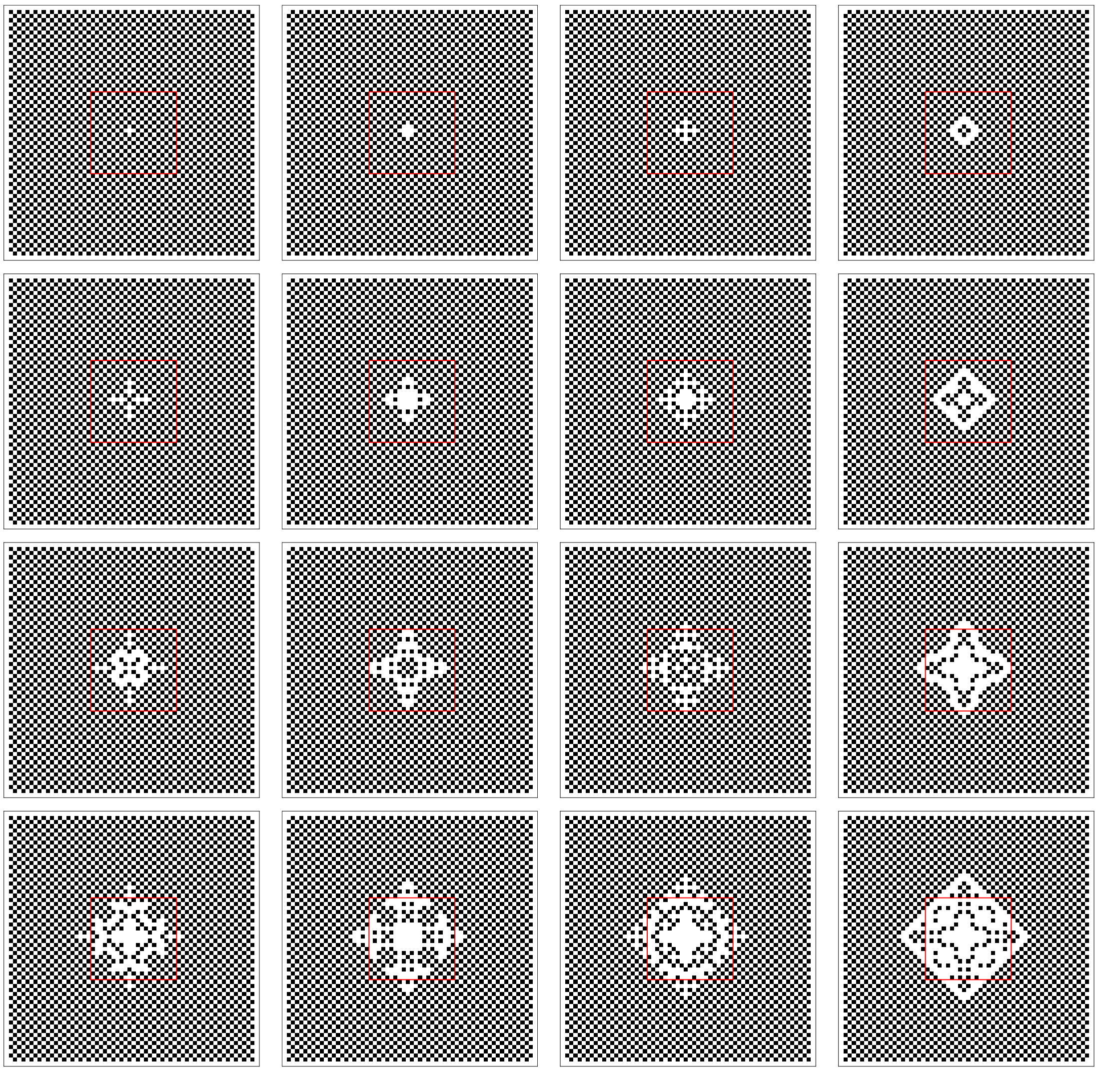
Here are the simplest cyclic backgrounds.



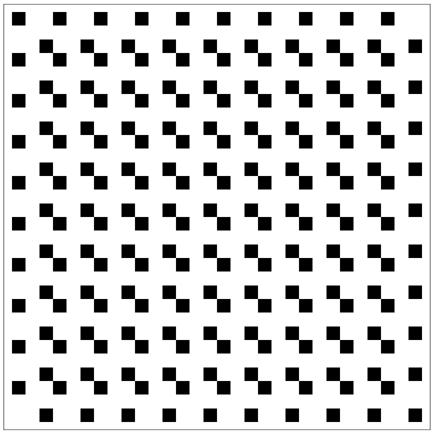
The more interesting cyclic backgrounds begin at size 5, but it makes sense to start with the simplest ones.

Recall that on the white background, bounded regions stay bounded, but there are plenty of stationary structures. Looking, for instance, for moving localized structures, we have to go to other backgrounds.

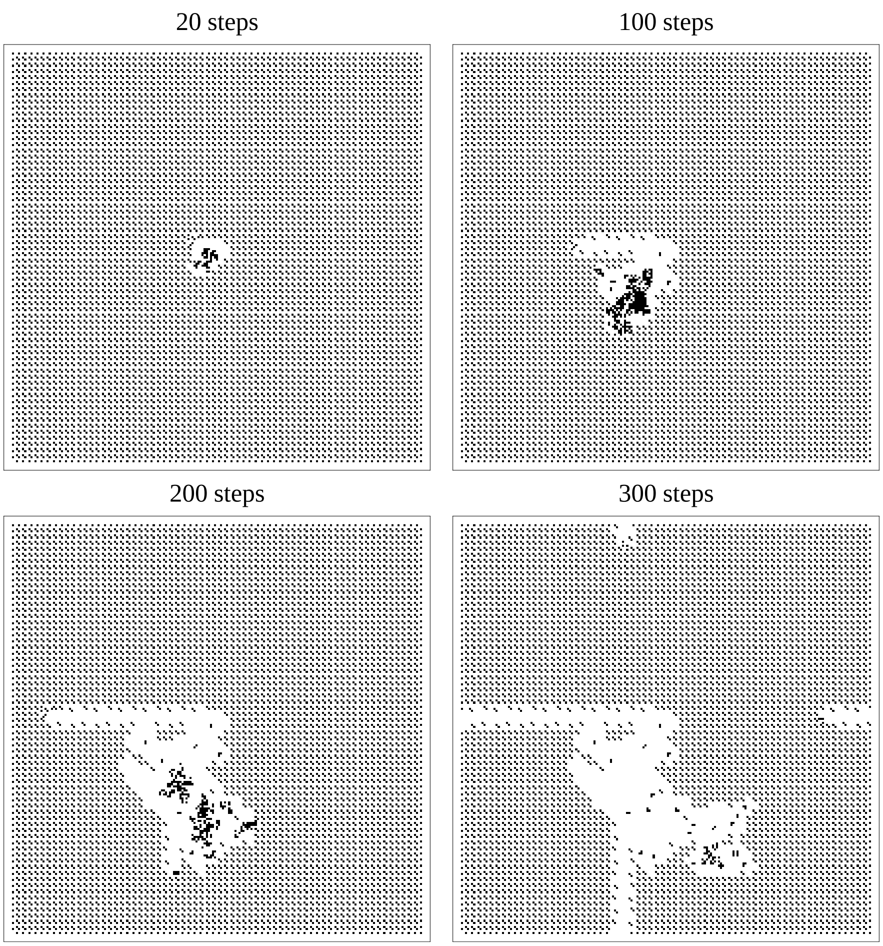
On many of the other backgrounds it is possible to get growing structures. The red square is to help see how fast it grows from a singular cell on a checkerboard grid. These are typical of 2D CA's, as in some of the pictures in NKS.



On this background,



there is even a moving localized structure. It leaves behind a trail. It begins here in a random tangle.



The structure's leftover tail has the property of forming walls which stop other structures.

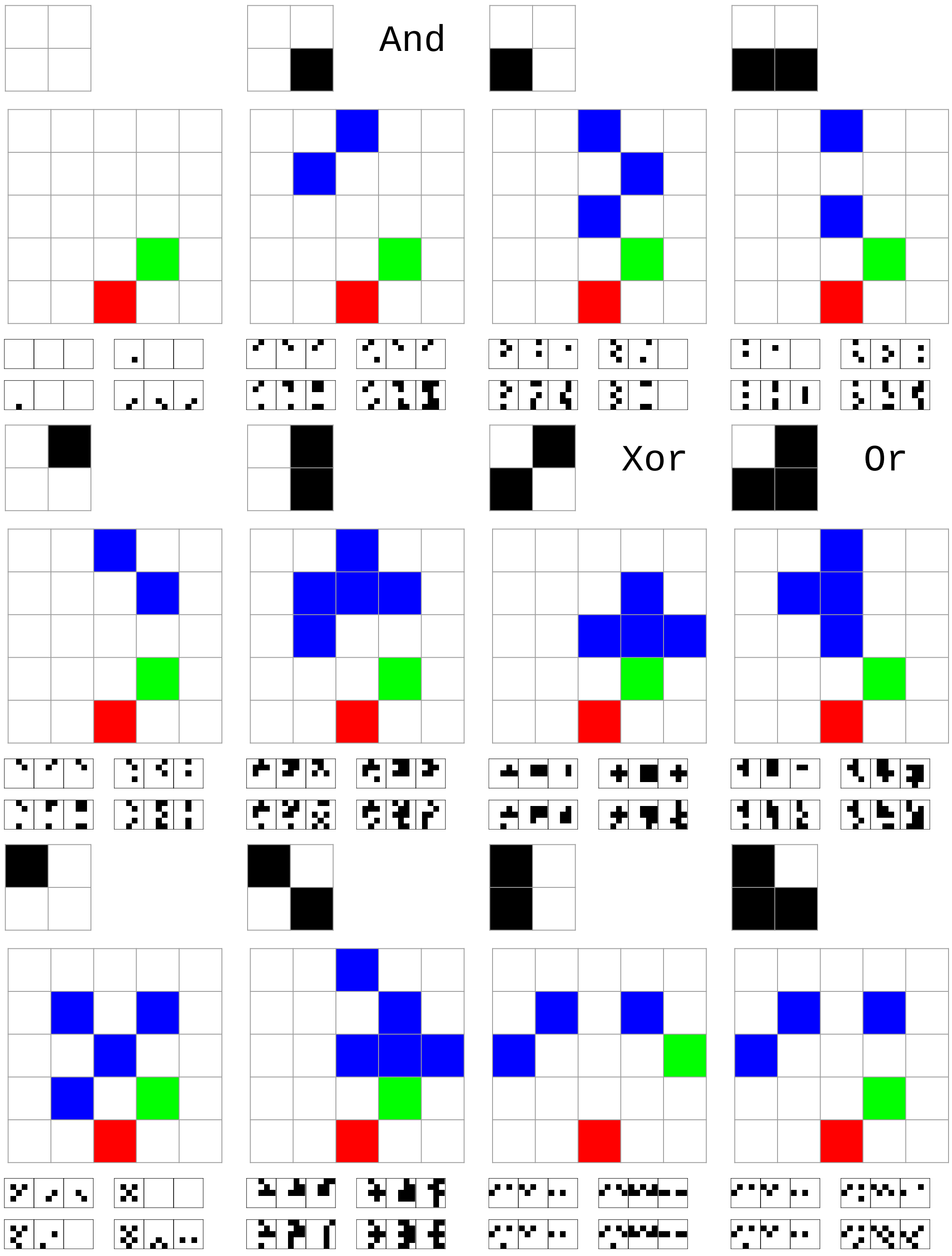
Logic

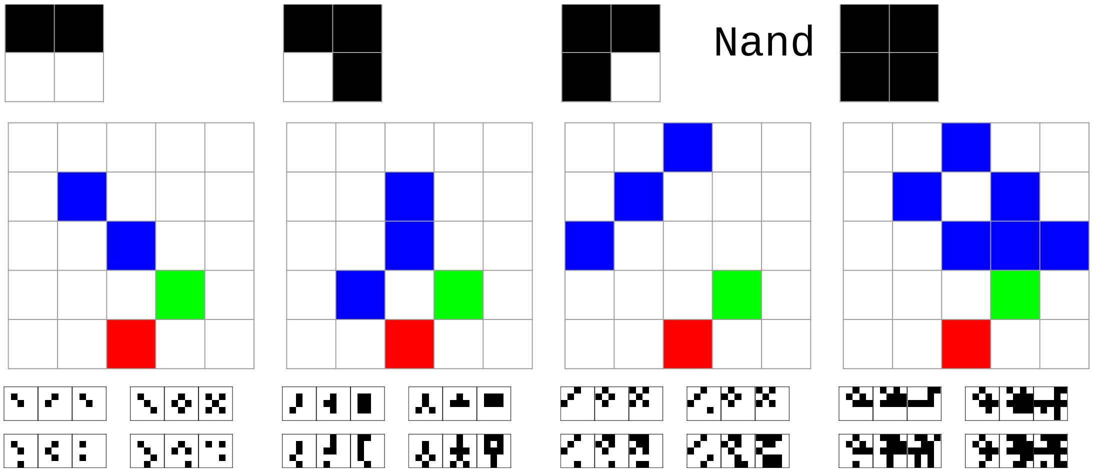
Moving onto the cases of special initial conditions, and starting with finite computations, there is the question of boolean expressions.

What kind of finite computations are supported? It seems like it is fairly easy to find initial configurations which perform any specific finite computation.

For instance, here are the boolean operators. One asks whether the center cell is white or black after two steps.

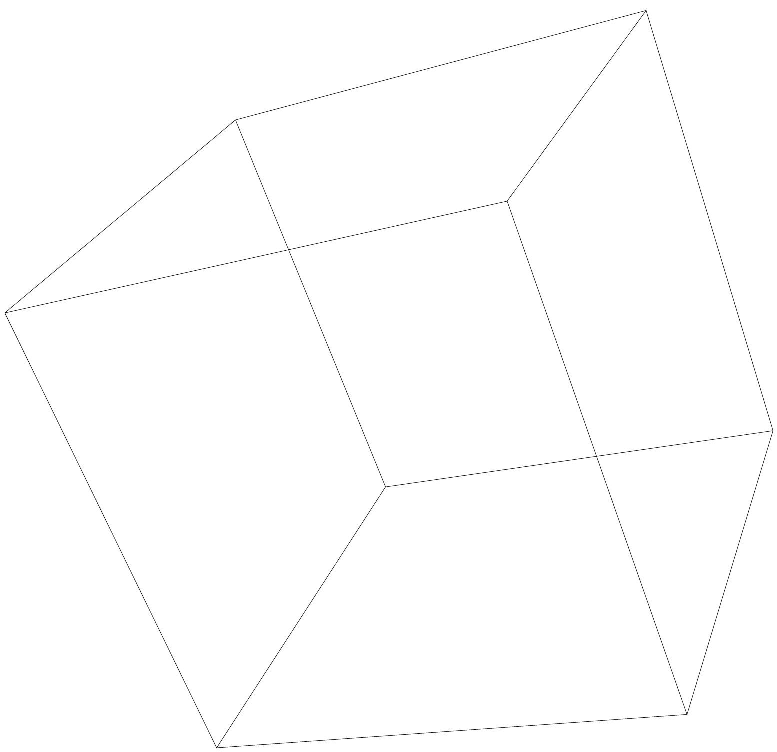
The blue cells are fixed as black, the green and red cells are the two inputs, and the output is the center cell after two steps. The top row shows the boolean operator, the grid with the colors shows the inital configuration, and the bottom row shows the two-step evolutions in the four cases.



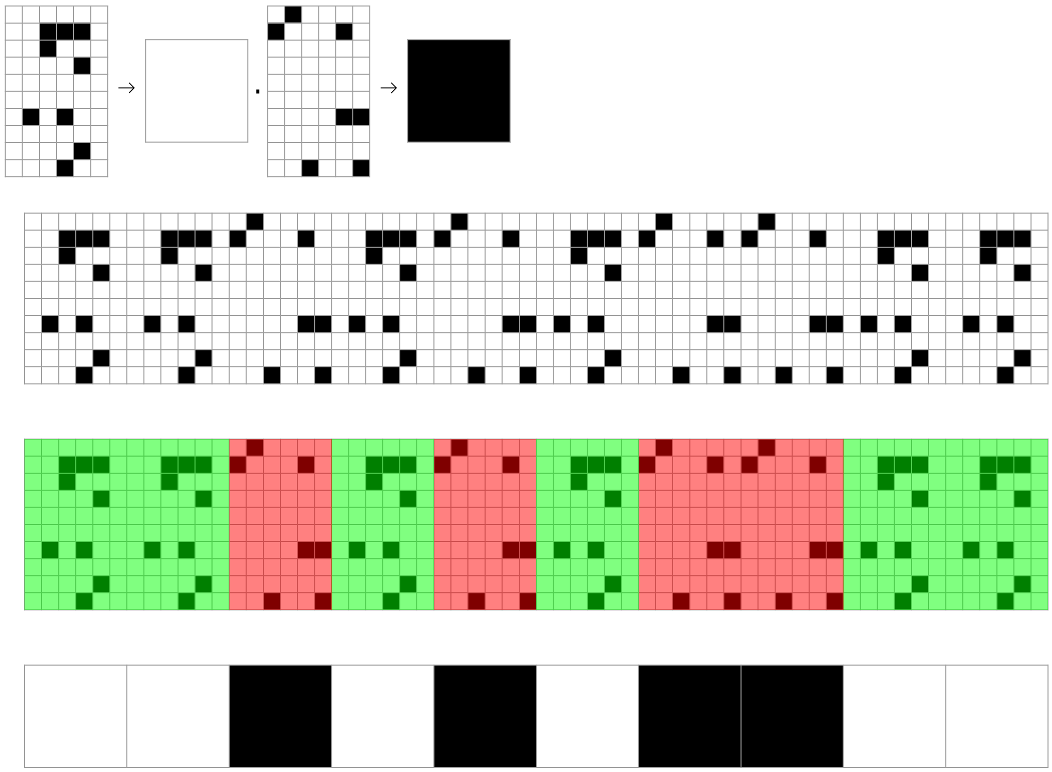


Emulations of one dimensional ECA

There are a few obvious ways to emulate a 1D CA with a 2D rule.

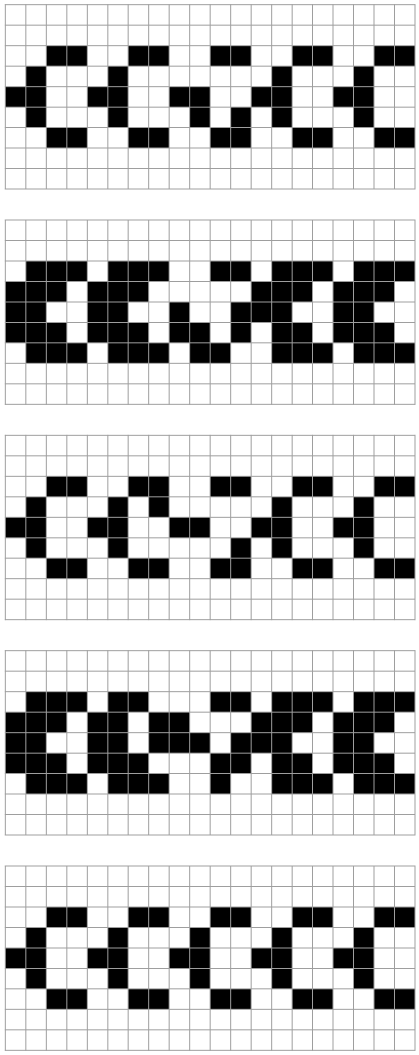


One can consider block emulations on a 2D cyclinder. Each band corresponds to white or black.

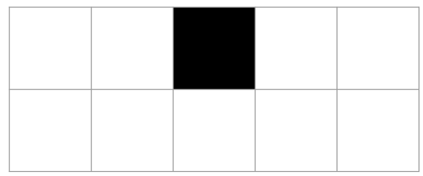


Because code 52 stays within its boundaries, it is also possible to have block emulations on a strip bounded by white cells on the infinite grid.

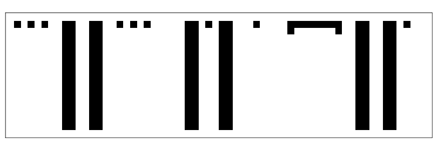
Here is rule 72.

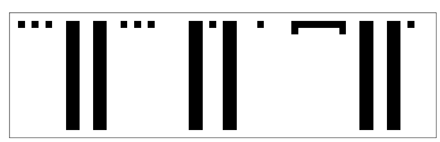


height 7
width 4



rule 72
ww–strip





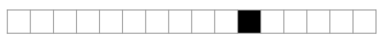
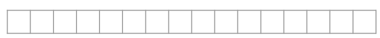
There is a further possibility of having black cells in the lower half. Having black cells in both halves is taken care of by symmetry since code 52 is the same when white is switched with black.

In addition to block emulations, where one block is white and another block is black, there is also the coarse-grain emulations, and also the coarse graining of a subalgebra.

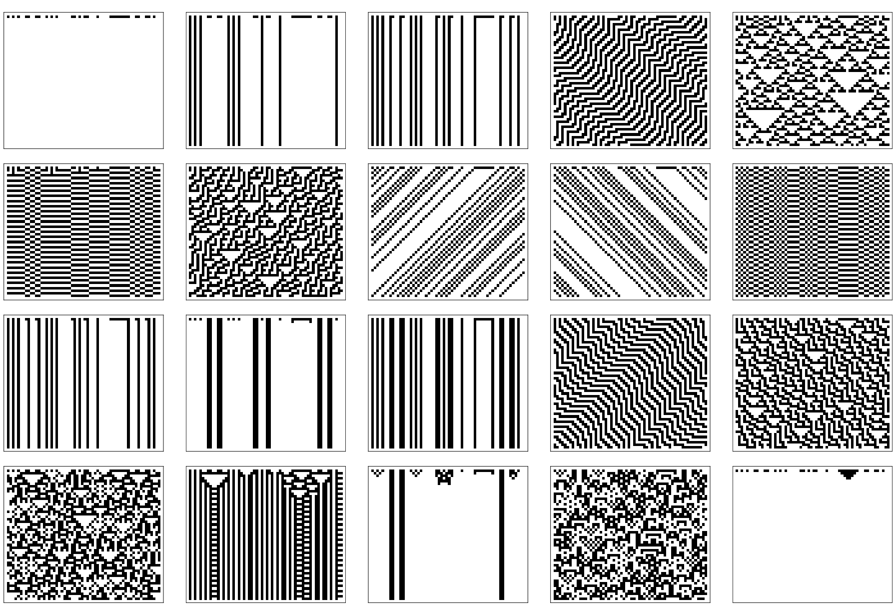
In all, I was able to find emulations for 65 out of the 255 ECA's. They were all found by brute search. By comparison, one of the ECA with a typical number of known ECA block emulations is rule 41, which can do these 11 other rules [NKS, p.692]

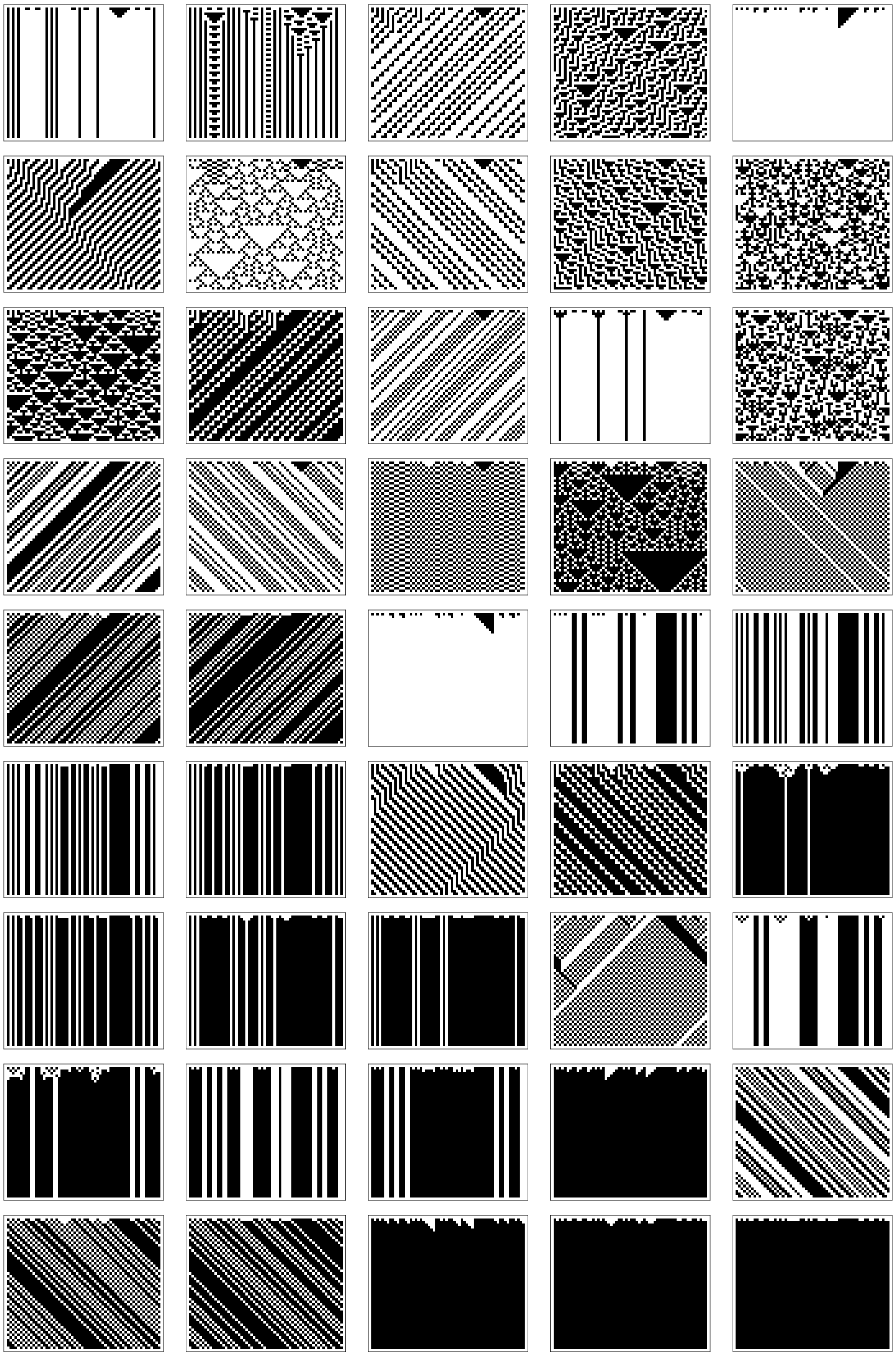
Range	ECA rules emulated by code 52
0 to 15	■□□□■□□□□□□□■□□■
16 to 31	□□□□□□■■□□□□□□■□
32 to 47	□□■□□□□□□□□□□□□□
48 to 63	■□□■□□□□□□□□□□□□
64 to 79	□□□□■□□□■□□□■□□□
80 to 95	□□□□□■■□□□■□□□■□
96 to 111	□□□□□□□□■■□□□□□□
112 to 127	□□□□□□□□□□□□□□□□
128 to 143	■□□□■■■■■□□□□□■□
144 to 159	□□■□■■■■□□□□□□■□
160 to 175	□□■□■■□□□□■□□□□□
176 to 191	■□■□□□■□■□■■□□□□
192 to 207	■□□□□□□□■□□□■■□■
208 to 223	□□□□■□■□□□■□□■■■
224 to 239	□□■□□□□□■■□□■■■□
240 to 255	■□■■□□□□□□□□■□■■

```
  0    4   12   15   22   23   30   34   48   51   68   72   76
 85   86   90   94  104  105  128  132  133  134  135  136  142
146  148  149  150  151  158  162  164  165  170  176  178  182
184  186  187  192  200  204  205  207  212  214  218  221  222
223  226  232  233  236  237  238  240  242  243  252  254  255
```

Range	ECA rules emulated by ECA Rule 41
0 to 15	
16 to 31	
32 to 47	
48 to 63	
64 to 79	
80 to 95	
96 to 111	
112 to 127	
128 to 143	
144 to 159	
160 to 175	
176 to 191	
192 to 207	
208 to 223	
224 to 239	
240 to 255	

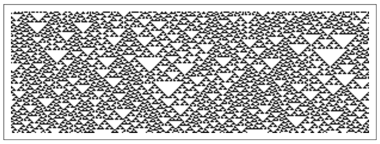
This picture summarizes what those emulated rules do from random initial conditions



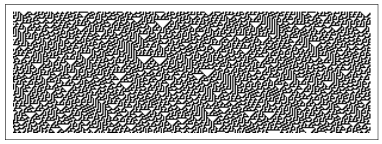


To me, the most interesting rules it can emulate are 22, 30 and 94, both of which can perform nested and complex behavior. Other notables include 90 and 184.

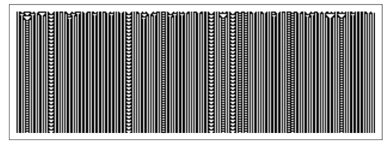
22



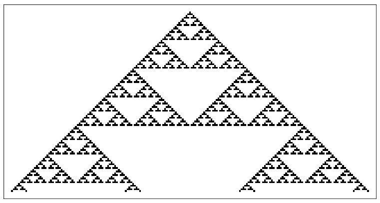
30



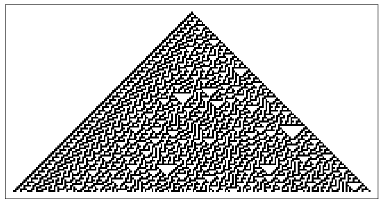
94



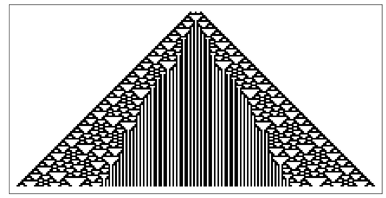
22



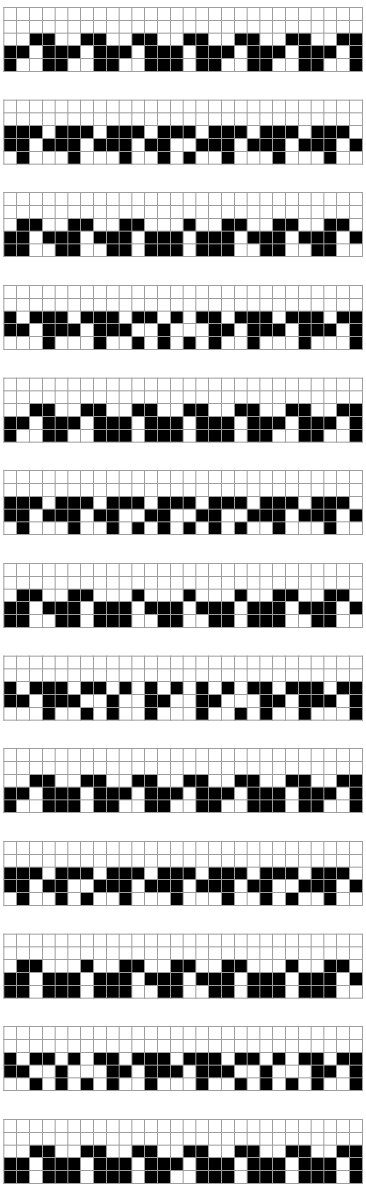
30



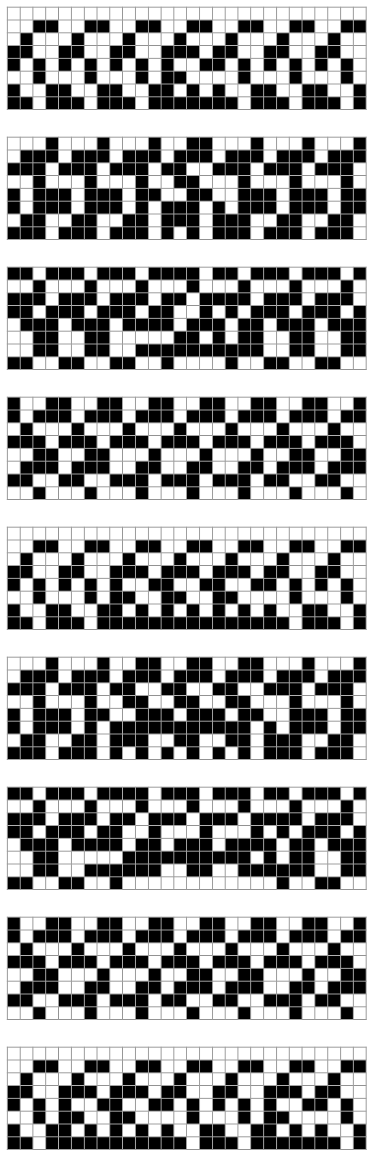
94



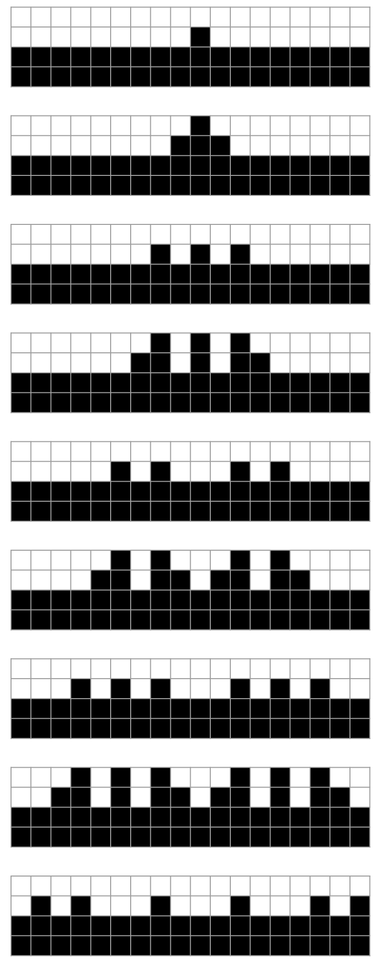
3 steps of rule 22



2 steps of rule 30



4 steps of rule 94



Here we see those rules in typical initial conditions, to get a sense for what this means for the range of behaviors code 52 in special initial conditions.

[[diagram too big??]]

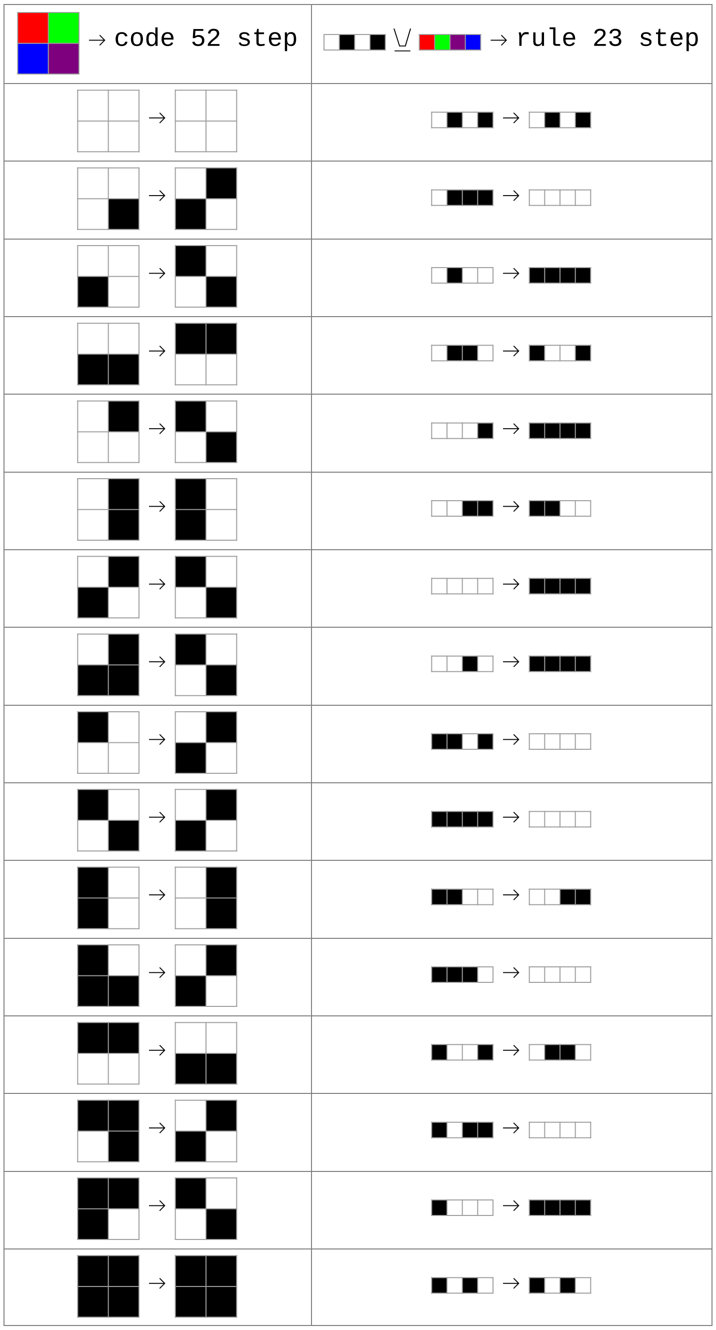
Going back to the possible behaviors along the white-black boundaries, where rule 150 was typical, and rule 90 was rare, we can estimate that the next most common rule would be XXX. Occuring from a purely stochastic process, we can calculate the odds of finding it completely randomly on a strip of size XXX would be about XXX.

One reason this is interesting is that there is often in nature the situation where something happens extremely rarely. For instnace, errors in the process of DNA replication, or certain radioactive decays. Here we have something which is definitely, easily observable, happening completely deterministically, even in a relatively small system.

Emulations of code 52

I would have liked to say that I found some interesting emulations of code 52 with other rules. That would be a nice way to help bring rules together, as predicted by the PCE. But nothing too interesting was found.

Here is rule 23 emulating the 2by2 grid of code 52. It only works for this size, so is not a true emulation of code 52. It works by lining up the cells as follows and then reversing the colors on the even squares.



Conclusions

Code 52 has a variety of behaviors, not just in the generic situation, but also in special initial conditions.

It seems that the technology for analyzing and emulating behaviors is increasing in power. I expect that it won't be long until this rule and many others are shown to be universal, in relatively straightforward ways,