Cellular Automaton Performing Two-Coloring of Square Tiled Planes

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Motto

It does not matter how good you are in it. The NATURE does it always better!

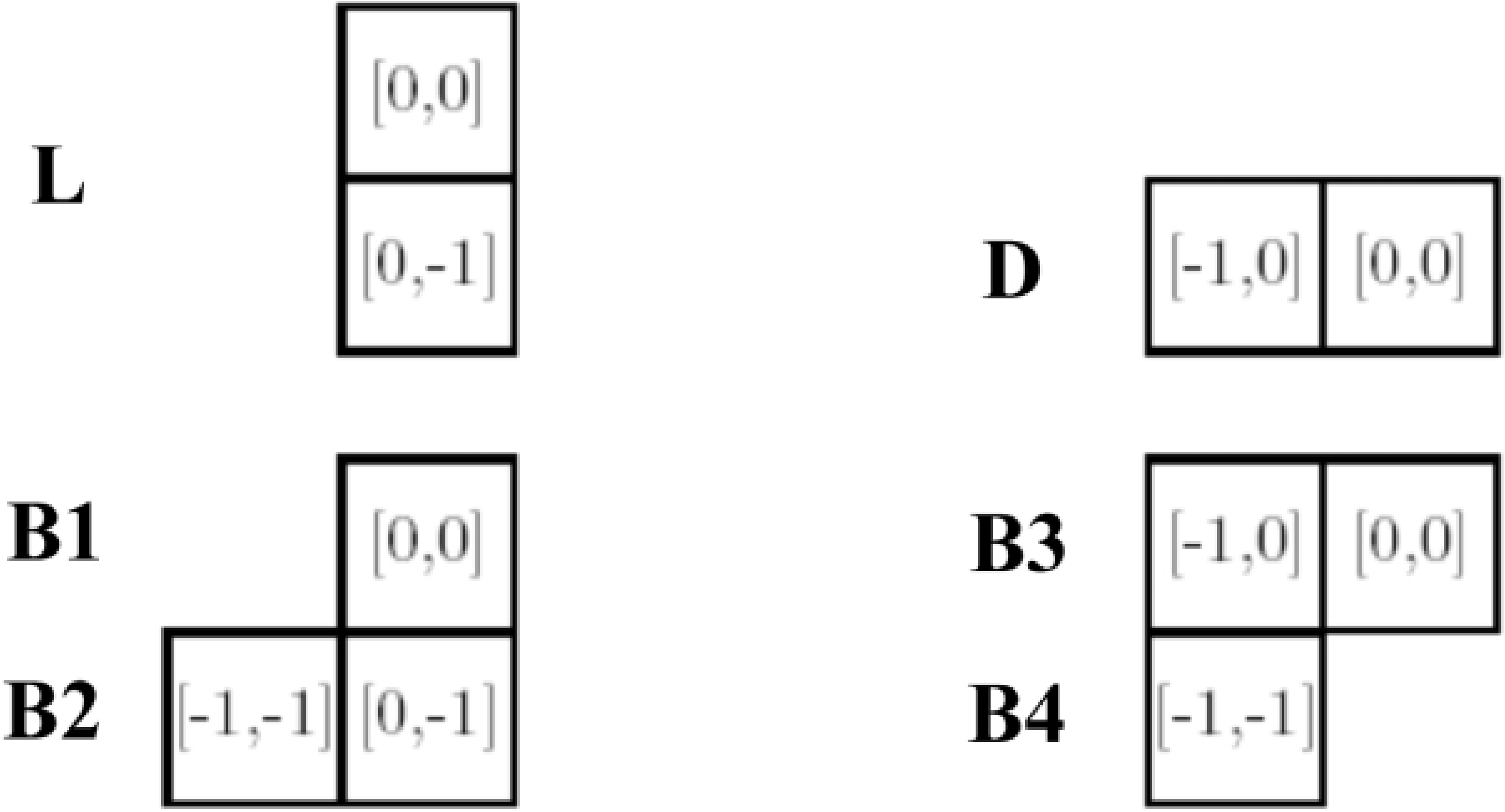
Outline

- Motivation
- Deterministic, two-colouring of square tiled planes with four neighbours
- Two-colouring in presence of three neighbours, a note
- The Penrose tiling – future work
- Future > towards the general rule
- Conclusions

Motivation

- The question is: “Could we construct a general deterministic rule performing fast colouring of tilings like the Penrose one?”
- The problem is studied on a regular square grid due to its simplicity (rem. mod (x+y))
- Could be the final algoritm used for the Penrose tiling?
- The three neighbour case as the way from the regular grid towards the Penrose case

Neighbourhoods



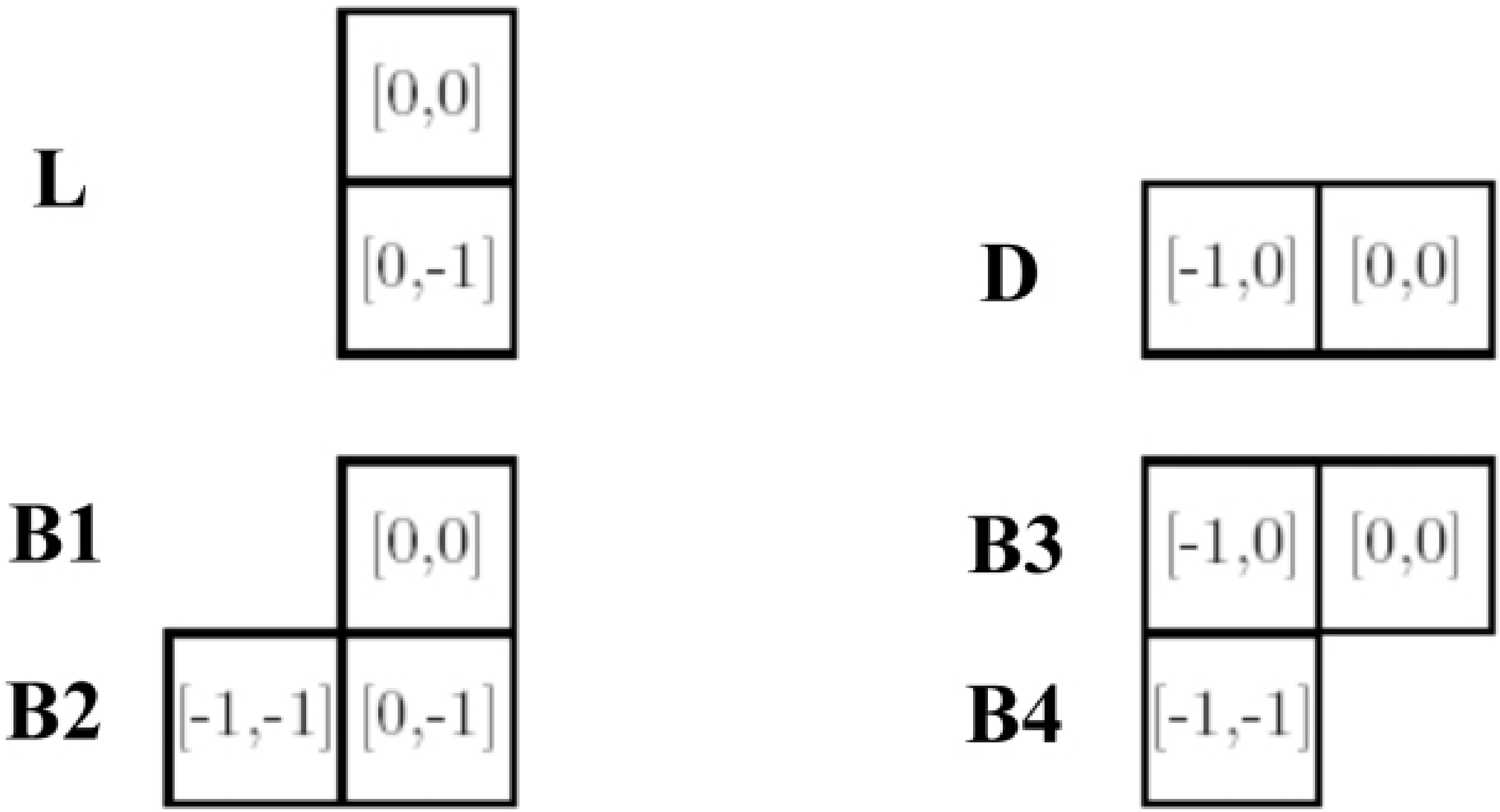
Rule **B1** - vertical

- **B1:** If *time*%4 = 0
  - (a) and if the colour of [-1,-1] is white, the colour of [0,-1] is black, and **y**% 2 = 0 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,-1] is black, the colour of [0,-1] is white, and **y**% 2 = 0 then colour of [0,0] becomes black.

Rule **B2** - vertical

- **B2:** If *time*%4 = 1
  - (a) and if the colour of [-1,-1] is white, the colour of [0,-1] is black, and **y**% 2 = 1 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,-1] is black, the colour of [0,-1] is white, and **y**% 2 = 1 then colour of [0,0] becomes black.

Neighbourhoods



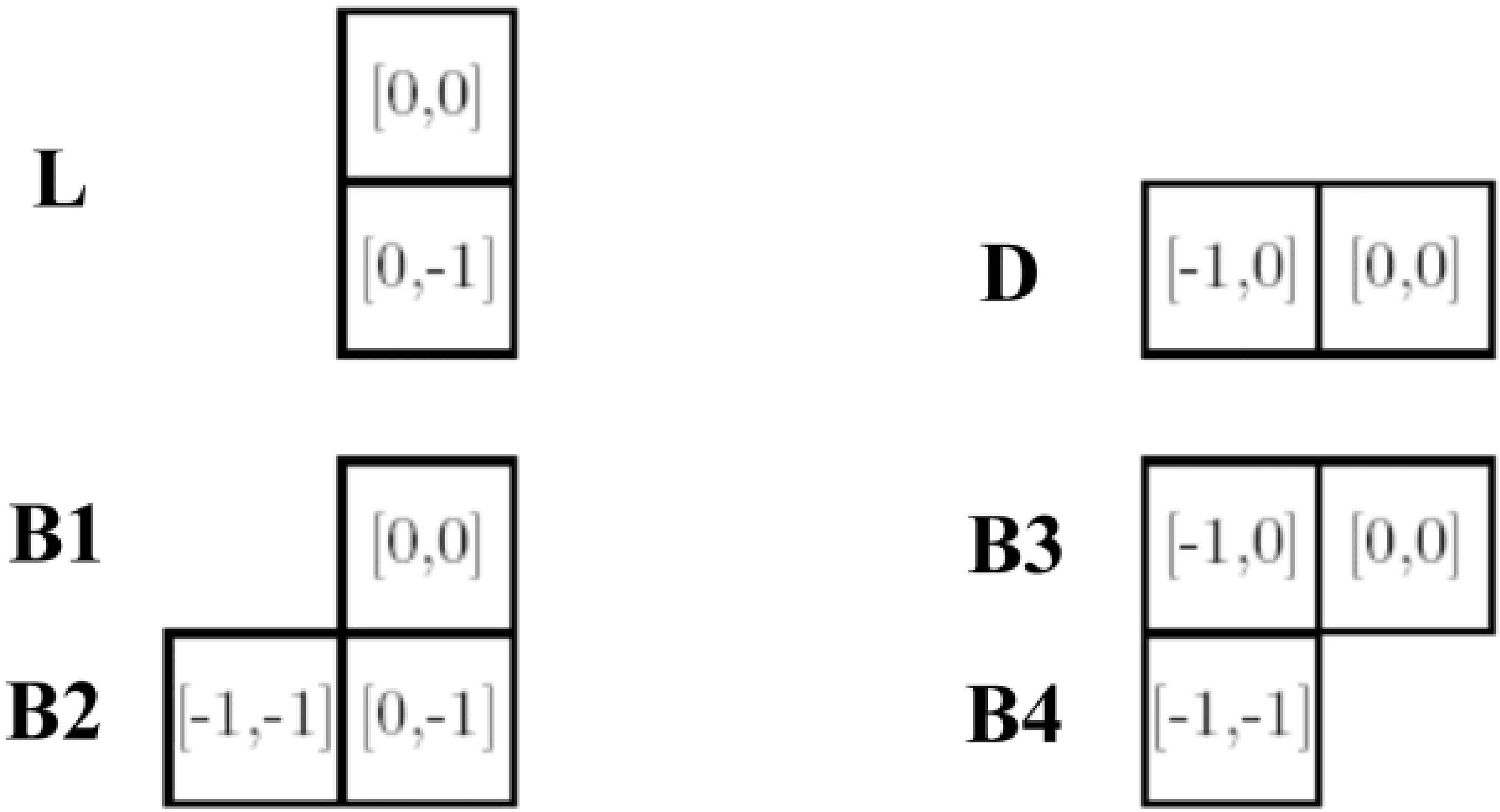
Rule **B3** - horizontal

- **B3:** If *time*%4 = 2
  - (a) and if the colour of [-1,-1] is white, the colour of [-1,0] is black, and **x**% 2 = 0 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,-1] is black, the colour of [-1,0] is white, and **x**% 2 = 0 then colour of [0,0] becomes black.

Rule **B4** - horizontal

- **B4:** If *time*%4 = 3
  - (a) and if the colour of [-1,-1] is white, the colour of [-1,0] is black, and **x**% 2 = 1 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,-1] is black, the colour of [-1,0] is white, and **x**% 2 = 1 then colour of [0,0] becomes black.

Neighbourhoods



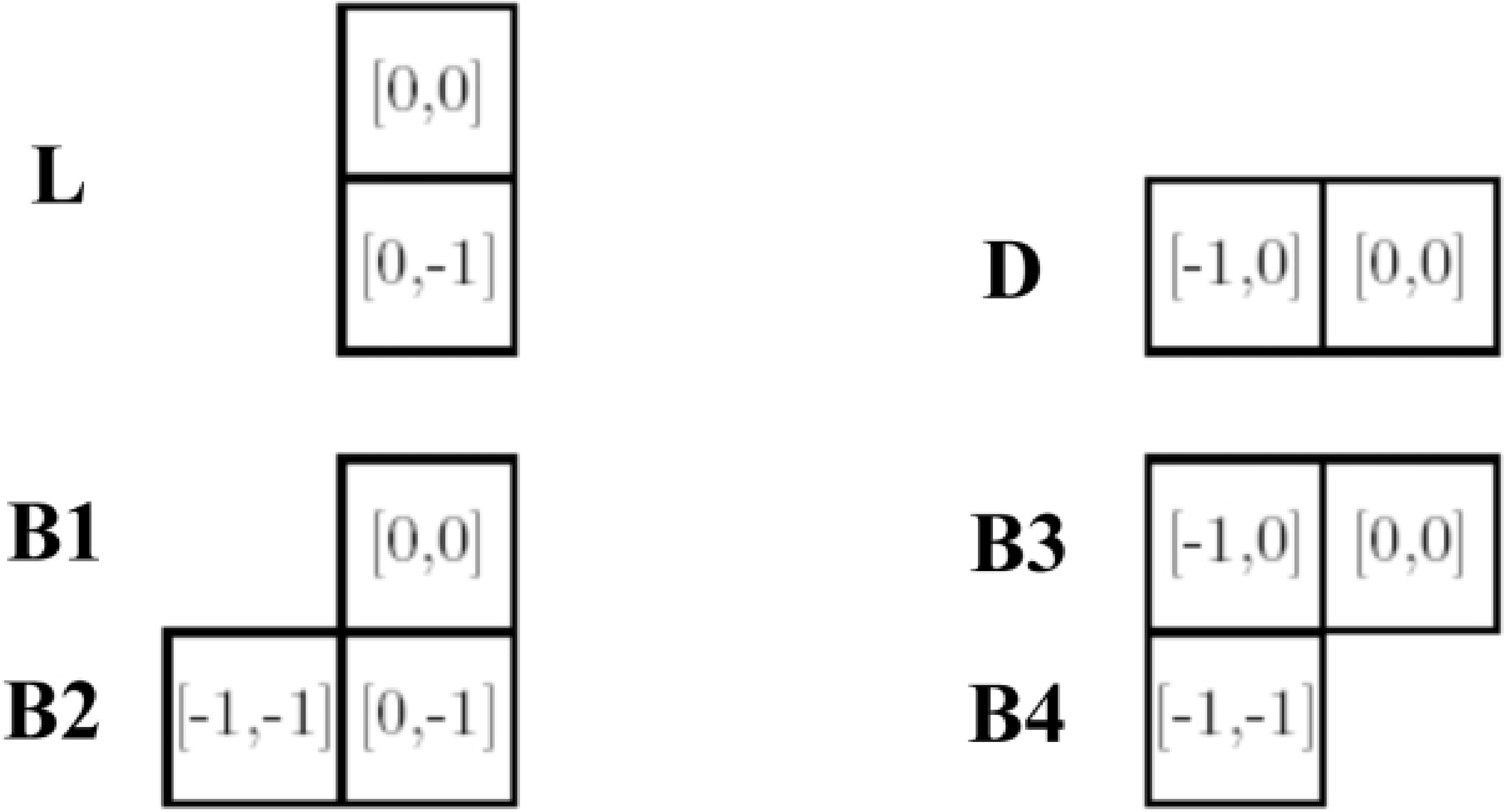
Rule **L1** – left edge

- **L1:** If *time*%2 = 0
  - (a) and if the colour of [0,-1] is black, and **y**%2 = 0 then colour of [0,0] becomes white,
  - (b) or if the colour of [0,-1] is white, and **y**%2 = 0 then colour of [0,0] becomes black.

Rule **L2** – left edge

- **L2:** If *time*%2 = 1
  - (a) and if the colour of [0,**-**1] is black, and **y**%2 = 1 then colour of [0,0] becomes white,
  - (b) or if the colour of [0,**-**1] is white, and **y**%2 = 1 then colour of [0,0] becomes black.

Neighbourhoods



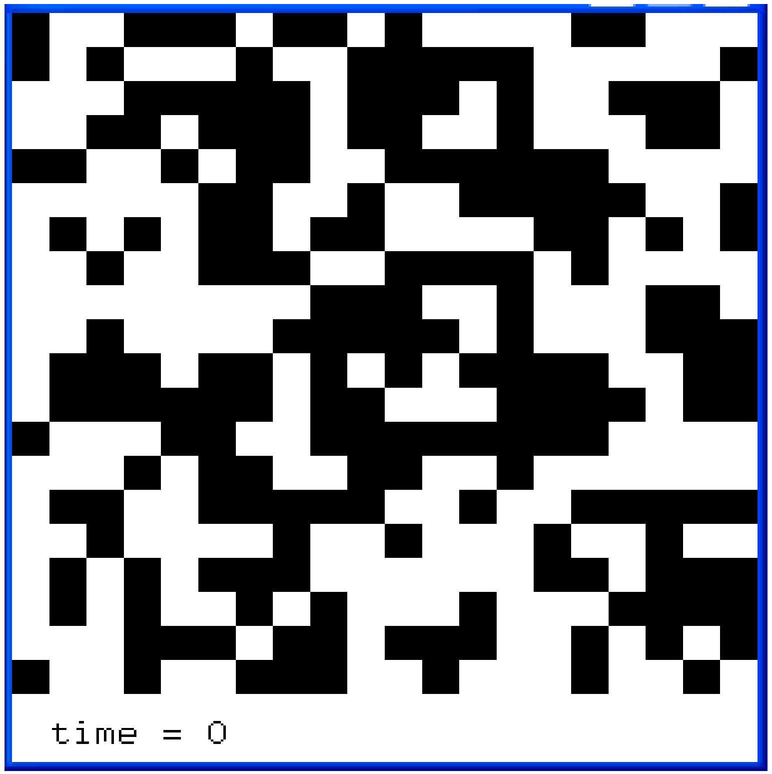
Rule D1 – bottom edge

- **D1:** If *time*%2 = 0
  - (a) and if the colour of [-1,0] is black, and **x**%2 = 0 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,0] is white, and **x**%2 = 0 then colour of [0,0] becomes black.

Rule D2 – bottom edge

- **D2:** If $time\%2 = 1$
  - (a) and if the colour of [-1,0] is black, and **x**%2 = 1 then colour of [0,0] becomes white,
  - (b) or if the colour of [-1,0] is white, and **x**%2 = 1 then colour of [0,0] becomes black.

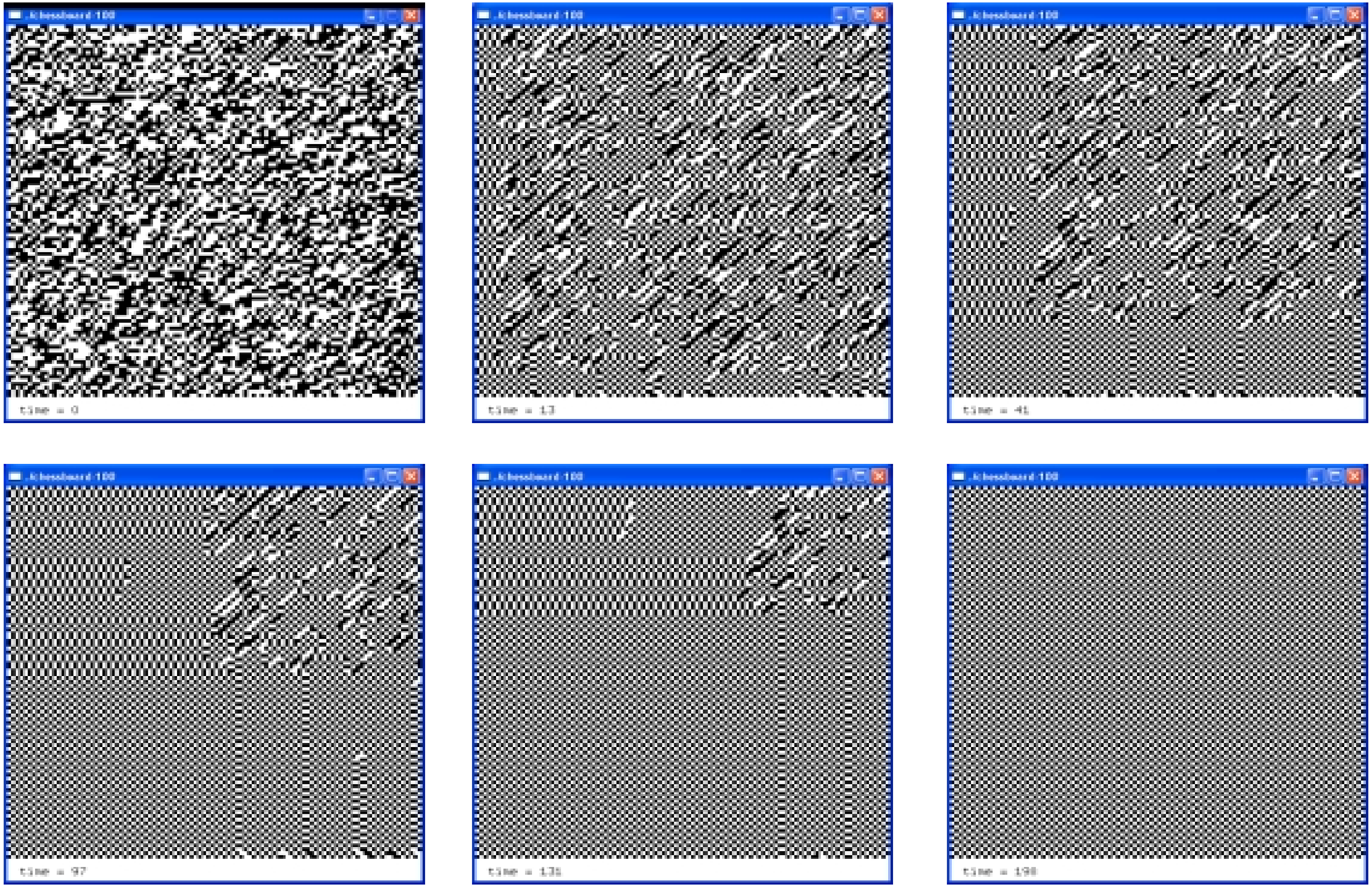
An initial configuration



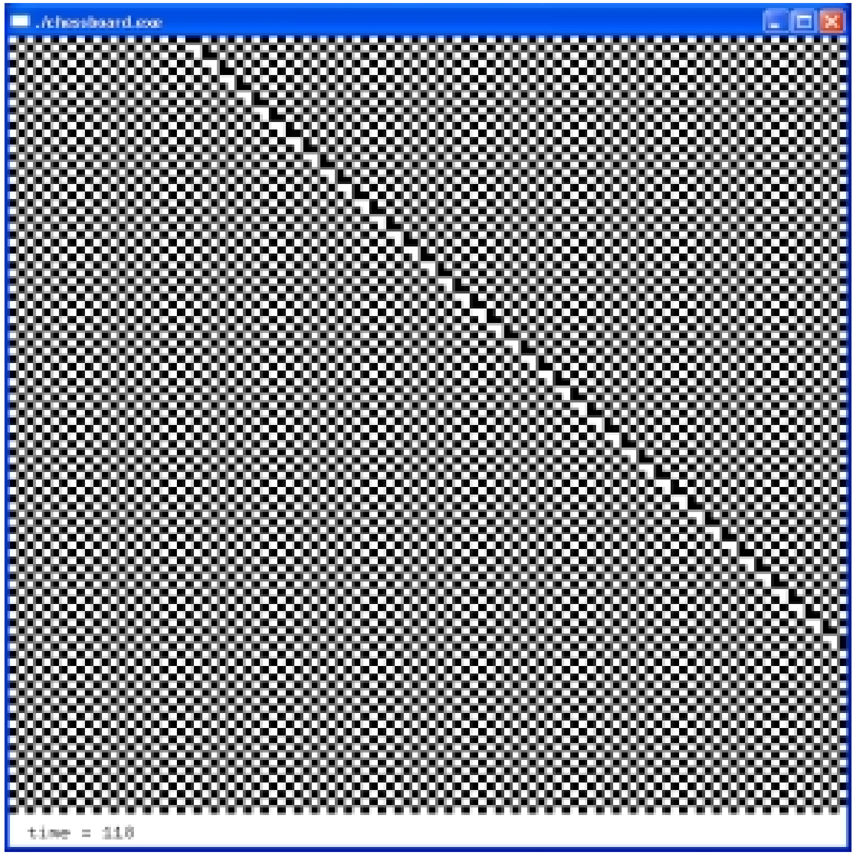
An random intial condition
of the world having size
20 x 20 cells is provided here
as a simple example.

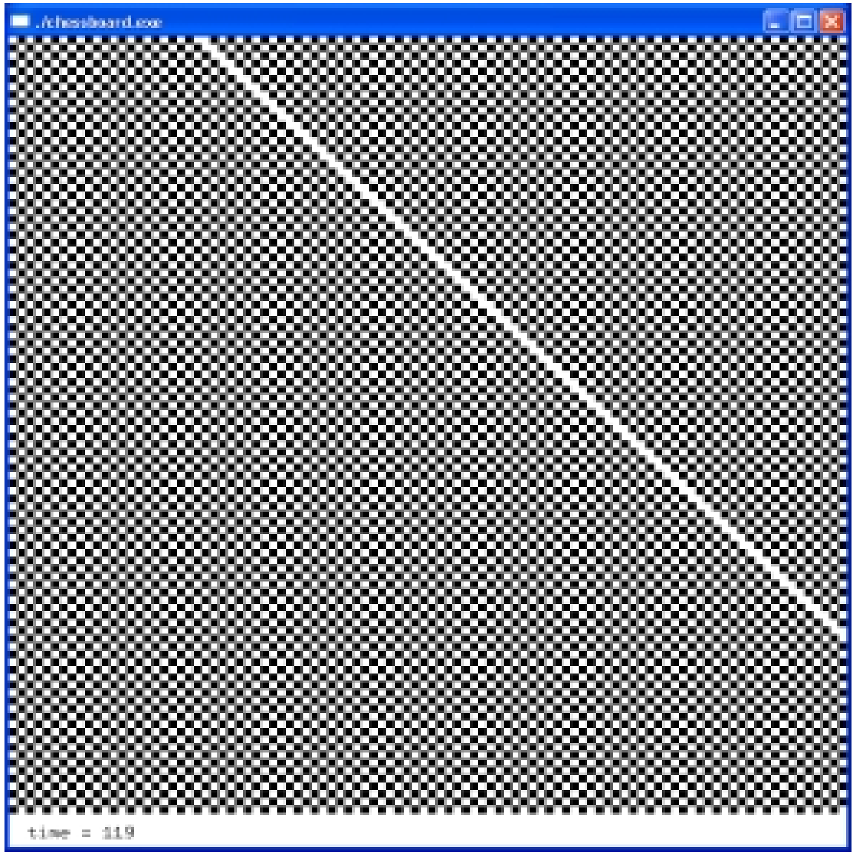
Evolution sequence - detail

Evolution sequence - random



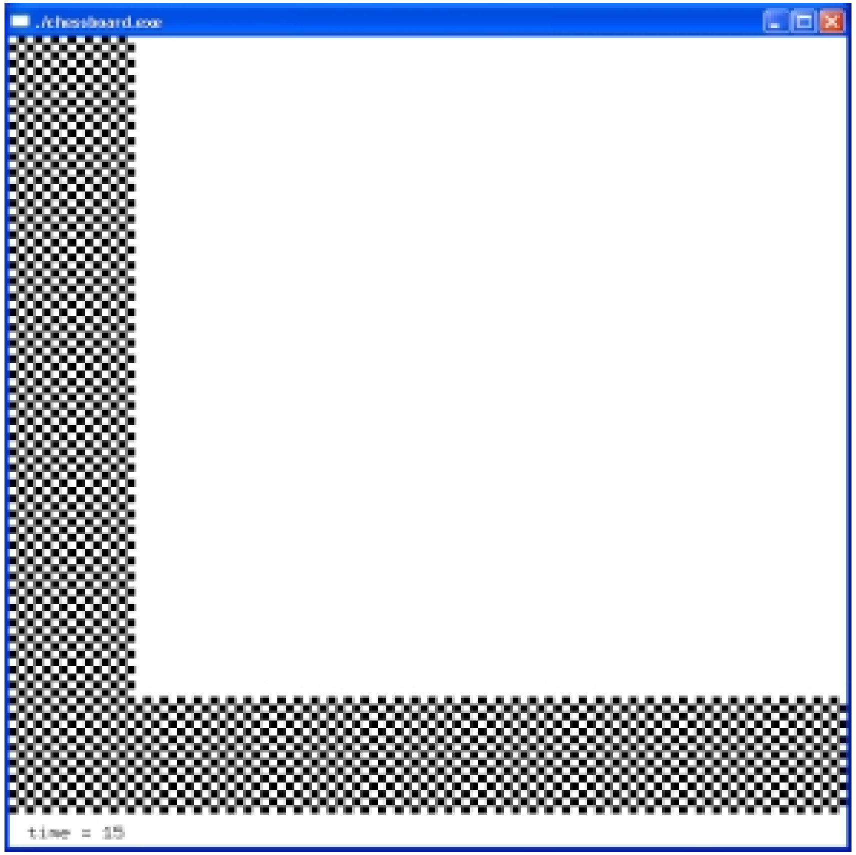
Antiphase separation border



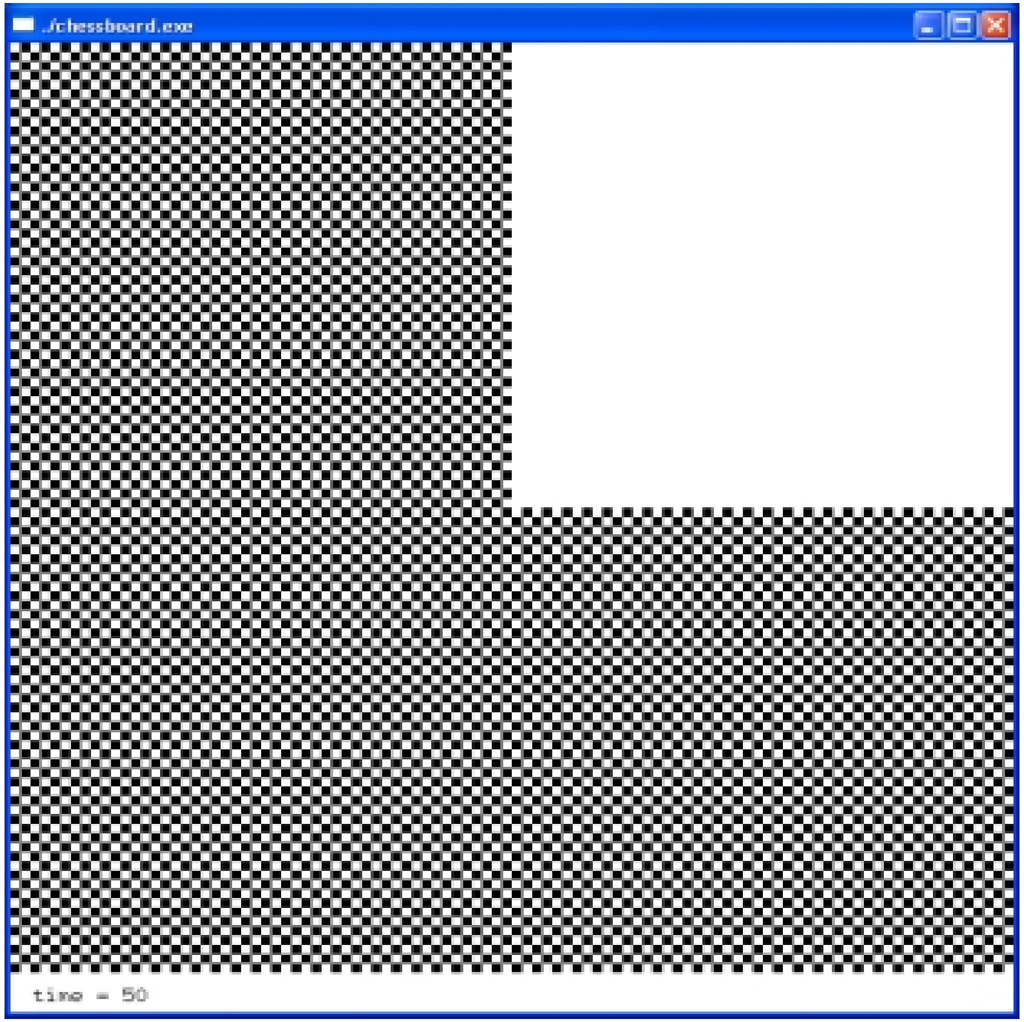


The total simulation time is 195 steps in this case!
A substantial time increase.

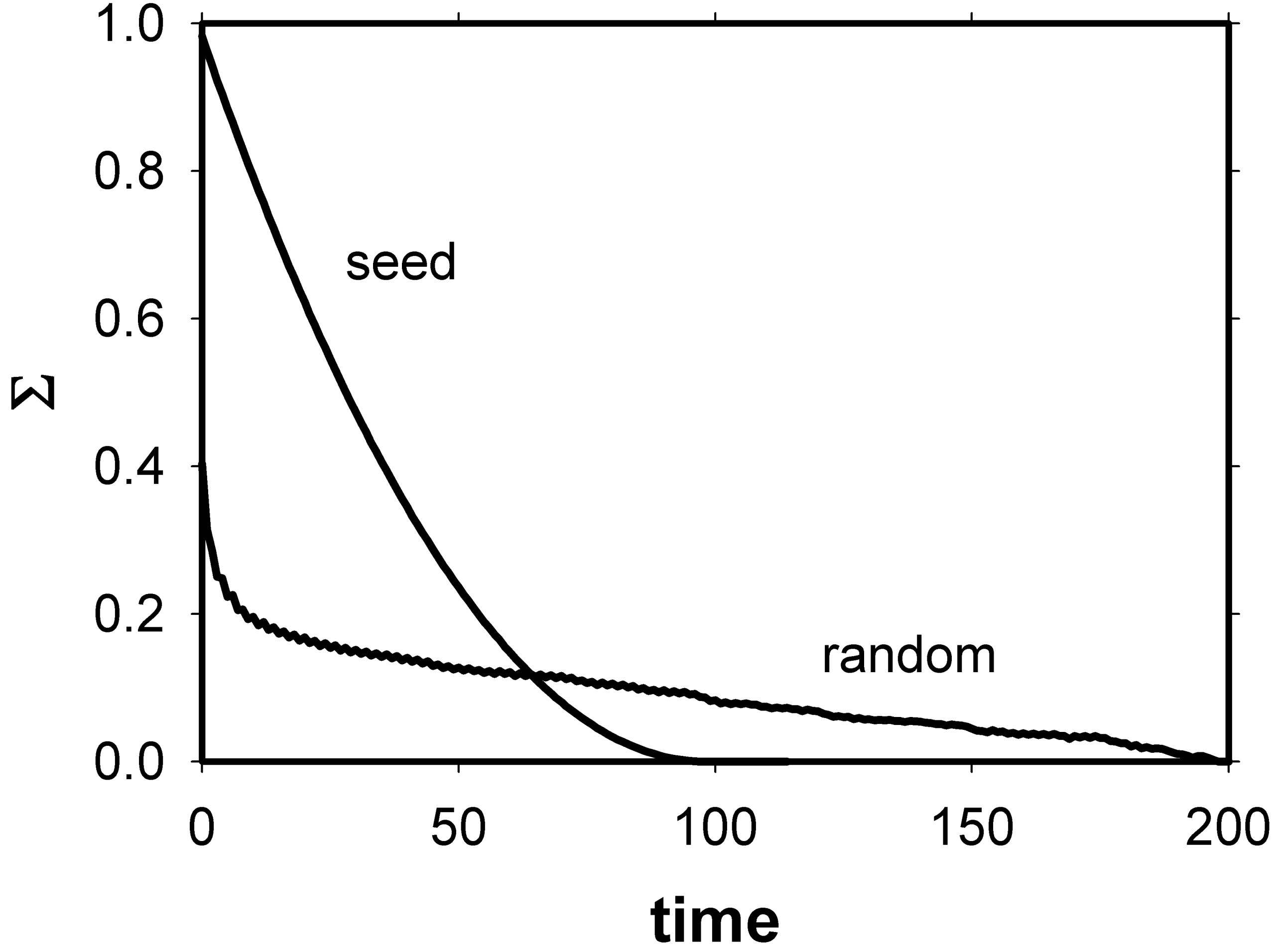
Evolution sequence - seed



Evolution sequence - seed



Disorganization vs time



Evolution of the disorganization level. The total incompatibility is equal to one.

Conclusions

- Two thing are important to fulfill simultaneously:
  - Find an efective rule
  - The initial conditions of the simulation
- It is not enough to find just an efective rule!
- It could leads to a tremendeous computational load in more complicated cases than this simple one presented here!

Conclusions - continues

- It obvious from this that although there is a solution of the problem the enormous number of attempts is necessary to start the correct process
- Hence, the work from looking for a rule is shifted towards the problem with 'waiting time' for having good luck with correct initial conditions
- Exactly as THE NATURE does!

Additional information

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