Exploring a Possible Relationship between Complex Behavior and Monotone Boolean Functions

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One of the important questions that has been asked about Cellular Automata is 'How is the behavior distributed in the rule space?' Of particular interest is to determine where in the rule space lies the complex behavior.

A new partition of the binary rules ($k = 2$) into two primitives (p_0 and p_1) has provided a means to explore their structure. Each rule in a rule space can be expressed as a linear combination of its primitives. Each of these primitives represents a boolean function and the resulting combination is the boolean function that defines the rule.

The two-dimensional mapping of the rules in the Elementary Cellular Automata rule space (ECA) shows a pattern of how the behavior is distributed. The complex rules are formed by primitives that belong to a particular subset of boolean functions. This subset is the Monotone Boolean functions and the complex rules are formed by primitives that belong to this set or their complements. In fact, each complex rule has one primitive that is a Monotone Boolean function and another that is a complement of a Monotone Boolean function.

The question is whether this relationship exists in rule spaces with a larger radius ($r > 1$). If this proves to be true there will be a link between the problem of enumerating all Monotone Boolean functions for a given number of variables and enumerating all rules with complex behavior for a given rule space.

Not all combinations of Monotone Boolean functions and their complements produce rules with complex behavior. There is a need to find the sufficient and necessary conditions for these functions so that a rule with complex behavior can be generated. There is a subset of primitives within the Monotone Boolean functions set that combined with a subset of all the complements of the MBFs that may produce rules with complex behavior.