Simulation of Traffic Regulation and Cognitive Developmental Processes: Coupling Cellular Automata with Artificial Neural Nets

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This paper deals with the advantages of so-called hybrid systems, that is, systems consisting of two or more different models. We present here two cellular automata coupled with artificial neural networks (neural CA). The first example is the model of a traffic regulating system. The cells of the CA represent different cars, and a certain neural net regulates the traffic. The second example shows a model of cognitive learning in dependency of social contexts. The cells of a CA consist of two types of neural networks. First results demonstrate the fruitfulness of this approach.

1. Introduction

For some time the nearly universal usability of cellular automata (CA) is well known and frequently demonstrated in many different applications in the natural and social sciences. Famous examples are the CA model of social segregation by Schelling [1][1] and the improvement of the biochemical hypercycle by Boerlijst and Hogeweg [2] using an appropriate CA model. The mathematical reason for this usability is the equally well-known fact that CA are equivalent to potential universal Turing machines (cf. [3]), which means that in practice each computable physically real system can be modeled by a suitable CA model (see also [4]).

However, certain complex processes can only be modeled with difficult enlargements of the original CA logic. CA are a paradigm of self-organizing processes but the traditional CA models do not take into account the dependence of complex systems on their environment. In particular, it is rather complicated to model such important char-

[1]Schelling became Nobel laureate in 2005.

acteristics of complex systems like those of learning, adaptability, and flexibility. All these characteristics mean that the complex system is capable of changing some parts of its "structure," that is, its rules of transition and its particular topology [5]. To be sure, it is in principle possible to provide CA models *via* additional rules with these characteristics. Yet it is much more efficient—and elegant—to enlarge CA models by their coupling with other models, for example evolutionary algorithms or artificial neural networks. These types were developed in order to model adaptive processes like those of biological evolution or, in the case of artificial neural nets, ontogenetic learning processes. Such coupling of different models is called the construction of "hybrid systems" [5, 6].

Principally one must distinguish between two kinds of hybrid systems: On the one hand there is the construction of "vertically coupled systems." This term means that there are two different systems like, for example, a CA and a genetic algorithm (GA) [7]. The GA operates "on" the CA in the way that the GA, according to certain environmental demands, changes the transition rules of the CA. The CA may be called for obvious reasons in this case the "base system" and the GA the "meta system" (the GA is "above" the CA, which is the original meaning of meta).

On the other hand there is the possibility of constructing a "horizontally coupled system." In this case two different models cooperate in a form of "division of labor" and exchanging information. Such an information exchange can lead to variations in the structure of one or both systems. But because the two—or more—coupled systems are principally not "above" or "below" the other system(s) we speak of horizontal coupling.

In this paper we give two examples of horizontally coupled systems, that is, the coupling of certain CA with particular artificial neural nets (NNs). These special kinds of hybrid systems may also be called neural cellular automata (neural CA).

2. First example: Simulation of traffic flows by cellular automata coupled with Kohonen feature maps

Imagine the following everyday situation: One is driving during the rush hour on a highway with several lanes, eager to come home. The traffic is already rather dense and the probability of backups increases, caused by inattentive drivers or anxious beginners. In addition, on the access roads to the highway are a lot of other drivers trying to enter the highway. In particular, some ruthless drivers seek gaps in the traffic and try to push their way into these gaps, increasing the probability of accidents. It is only a question of time that some of these drivers will cause a crash and the whole traffic will come to a stop.

In conurbations these situations belong to the normal problems of everyday life and the traffic authorities; for example, in Germany, have tried to regulate these traffic problems. Besides speed limits on the highways[2] there are special traffic lights on the access roads to the highways, which regulate how many cars may enter from each particular access road. For example, if the traffic is very dense, the green phase of the traffic light is very short so that only one car during this phase is allowed to enter. If the traffic is not dense; for example, during the late morning or on an early Sunday morning, the green phase is very long or the lights are even turned off. In cases of backups in front of the access roads the light will remain in the red phase until the backup clears.

The problem with this traffic regulating system is, as one can expect, that often the phases of the traffic lights are not well adjusted to the factual traffic flow. Sometimes the red phases are longer than the traffic situation needs. Sometimes the green phases are so long that too many cars cause just the situation the traffic lights should hinder. In particular, the variations of the light phases frequently are too slow: if the traffic situation changes very fast then the adjustments of the light phases do not occur fast enough. As these adjustments are regulated by automatic systems it seems obvious that these systems are not adaptive enough for fast changing traffic situations.

In order to improve this whole situation we developed a particular hybrid system or neural CA. This system at present is just a prototype. We intend to test it with real data, obtained by the traffic authorities in the Rhein-Ruhr Region (in the Western region of Germany), one of the largest conurbations in Europe, where such traffic regulating systems have been installed for some time.

The use of CA models for the simulation of traffic flows is basically not very new (e.g., [8]). Indeed, it is very obvious to use the basic logic of CA models for such purposes *via* the representation of single cars by single cells on the CA grid. Therefore, we just sketch our own traffic CA model by naming its most important characteristics.[3] The cells of the CA represent different types of cars, that is, different with respect to velocity and type of driving. These types, of course, do not change. The artificial cars move on different lanes of a highway. As in reality, because of the different velocities and types of driving, accidents and other problems will occur that lead to backups. In particular, a high density of traffic will increase the probability of accidents. In contrast to, for example, the model of Esser and Schreckenberg [8], our model additionally contains two lanes so that a faster car can overtake

[2]In Germany there is in principle no general speed limit on the *Autobahnen* (highways) but only in problematic regions where dense traffic is to be expected, or where there are dangerous driving conditions.

[3]The final model was implemented by Alexander Behme and Maik Buczek.

a slower one. Another addition is the insertion of access roads where new cars can enter the highway and other cars may leave it. The third most important addition is the introduction of traffic lights that regulate access to the highway. Here are the principal rules of the CA.

- If a fast car approaches a slower one from behind, then the faster one has to overtake the slower one, if that is possible; else, the faster car has to slow down, that is, to adapt its speed to that of the slower one. Before overtaking a slower car the fast car has to "look" whether or not the left lane is free. If a car remains in the right lane then it has fewer cells to observe than in the case of overtaking.

- If there are speed limits, all cars have to adapt their speed.

The last rule, of course, is not totally realistic because in reality there always will be car drivers who do not obey speed limits and other regulations. But to make the model as simple as possible we inserted only law abiding artificial citizens.

- If a backup is in front of a car, the car has to slow down and finally stop. If there is an obstacle in front of a car in the right lane the car looks if it can overtake, that is, if it can go to the left side. If that is not possible the car has to slow down and finally stop.

In more technical terms the CA is characterized the following way: The state of the cells is defined as $S = \{0, 1, 2\}$, with 0 = speed up, 1 = overtake, and 2 = adapt the speed (including stop).

The geometry of the CA is defined by an "enlarged" Moore neighborhood, that is, two additional cells outside of the respective Moore neighborhood of a "car cell" are taken into consideration.

Here are the main rules of transition.

- If the cell is in the right lane, then consider only the cells at the front, on the left, and the additional two cells on the left side.

If a car cell is in the right lane, then compute the state S_{t+1} as follows: R is the set of all empty cells in the right lane and r_i is an element of R.

- If the sum of the empty cells before a car r_i is greater than four, then the car speeds up, according to its type and possible speed limits.

L is the set of all empty cells in the left lane and l_i is an element of L. If one cell in the right lane has to overtake a slower car then the cell has to observe the left lane according to the following rule.

- If the sum of all empty cells l_i is greater than four, and if these empty cells are the adjacent cells, then the car can overtake. "Adjacent" means the cells immediately on the left side of the car, the two cells directly behind this cell, and one cell in front of this cell.

In more mathematical terms we obtain, when omitting the additional geometrical conditions:

$$s_{ir}(t+1) = \begin{Bmatrix} 0, & \text{if } \sum_{(i,r)\in R} s_{kr}(t) > 4 \\ 1, & \text{if } \left(\sum_{(i,r)\in R} s_{kr}(t) \leq 4 \wedge \sum_{(i,l)} s_{i,l}(t) > 4\right) \\ 2, & \text{if } \left(\sum_{(i,r)\in R} s_{ir}(t) \leq 4 \wedge \sum_{(i,l)} s_{i,l}(t) \leq 4\right) \end{Bmatrix}. \tag{1}$$

Frequently the highways in conurbation areas are overloaded. Therefore, in Germany (and probably in other countries too) the access roads to the highways are regulated by special traffic lights, as mentioned earlier. The traffic lights show, as usually, green, red, or yellow. It is possible to vary the length of the color phases, dependent, of course, on the density of the traffic at a certain time. The assumption in our model is that measurement stations exist, for example, 1 km before and after an access road.

In our CA model the traffic lights are regulated by a Kohonen feature map (KFM), which belongs to the type of unsupervised learning nets. The net is trained to perceive certain critical values of traffic density. This is done by inserting into the KFM several "prototypes" of traffic density; for example, medium density, high density, low density, and backups (Table 1).

If the KFM receives a certain density as input, then the network systematizes this information by placing it into the neighborhood of the most similar prototype. In other words, the KFM is trained by inserting the prototypes, which are placed on a grid. The information about different traffic densities will be "clustered" around these prototypes, according to the similarity of the new information to the prototypes. The KFM, so to speak, translates the similarities of certain vectors into spatial relations.

		Traffic light off	Normal phase of traffic light	Traffic light flashes
Distance (in m)	$x > 45$	X	–	–
	$45 \geq x > 25$	–	X	–
	$x \leq 25$	–	–	X
Speed (in km/h)	$x > 90$	X	–	–
	$90 \geq x > 50$	–	X	–
	$x \leq 50$	–	–	X
	$x < 20$	X	–	–
Number of cars	$20 \leq x < 33$	–	X	–
	$x \geq 33$	–	–	X

Table 1. Example for the measurement of the numbers of cars, the distance, the speed, and the corresponding phase of the traffic light.

Because the KFM belongs to the more difficult types of artificial neural nets and for the reason that usually only experts in neuroinformatics are acquainted with KFM models (there are different versions) we give some information about the basic algorithms of this KFM. More technical details can be looked up in any textbook on NNs. By the way, in our second example below we also use a KFM.

The learning rule is the so-called winner-take-all function, which is in the most general form:

$$\begin{aligned} w_{ij}(t+1) &= w_{ij}(t) + \epsilon\left(x_i - w_j(t)\right) \\ \left\|x - w_z\right\| &= \min_i \left\|x - w_j\right\| \end{aligned} \tag{2}$$

x is the input pattern and w_z is the "winning" neuron, that is, the neuron on the KFM with the minimal distance to the input pattern. ϵ is a learning rate, that is, a value which influences changing the weights.

The amount that the units learn will be governed by a neighborhood kernel h, which is a decreasing function of the distance of the units from the winning unit on the map lattice, the so-called Mexican hat function:

$$h_{iz} = \exp\left(\frac{-(j-z)^2}{2} * \sigma_z^2\right), \tag{3}$$

if $j - z$ is the distance of neuron j to the kernel and σ_z is the radius within which the units will be changed.

By inserting equation (3) into equation (2) we obtain the exact learning algorithm for our model by

$$w_{ij}(t+1) = w_{ij}(t) + \epsilon * h_{iz} * (x(t))w_{ij}(t), \qquad j \in \sigma. \tag{4}$$

For visualization purposes a screen-shot is shown in Figure 1. The system has run 126 time steps and the traffic has already become rather dense because of an obstacle. One traffic light is in the red phase, which means that no additional access is allowed from that particular access road.

Conclusion

The practical use of our system is the possible optimization of the factual traffic regulating system that already exists in different regions of Germany. Accordingly we plan to validate our model with real data obtained from the traffic regulation authorities and compare our system with the factual performance of the regulation system. In addition, we intend to propose the additional installment of NNs of our type into the automatic systems that regulate the variations of the light phases. By this the whole system should become more flexible, that is, adaptive to fast changing traffic situations. As we often have to drive on the

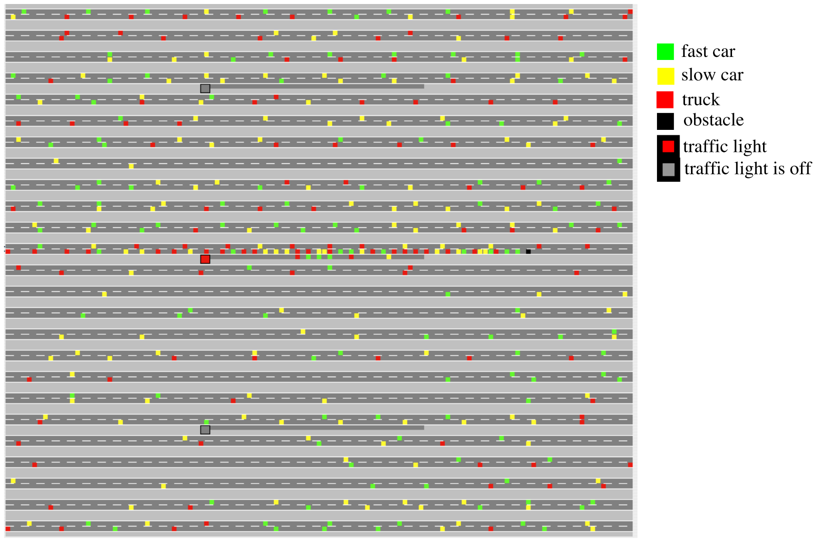


Figure 1. The second traffic light is in the red phase; the other two are off because no accidents or other obstacles disturb the traffic.

highways in the Rhein-Ruhr Region we are frequently irritated by some of the traffic lights. Perhaps our system will be able to make life a bit easier for many car drivers in this region.

3. Second example: The evolution of neural networks in a social context

Research in artificial intelligence (AI) has mainly been centered on the individual learning processes. As has been known in educational psychology for a long time, at least since the famous comparative studies of Piaget, the success of learning depends to a high degree on the social milieu of the learner's situation. Therefore, the modeling of learning processes *via* the use of AI systems must also take into account the essential role of the environment. Hence, by the concept "dependency of social context" we mean the cognitive development of a learning system that gets information and feedback from its environment and organizes its own evolution by constructing cognitive representations [9]. The factual development of the system is dependent on the one hand on its particular developmental logic, that is, the cognitive dynamics that govern its evolutionary path. On the other hand the environment or context respectively determines the development by orienting the system into certain directions *via* feedback, that is, evaluating the learning success, and by slowing or accelerating the whole process. In this section we describe a formal AI model that allows analyzing this "dynamical relationship" between ontogenetic processes and environmental feedback by computer experiments.

Referring to cognitive ontogenesis, the undeniable fact must be taken into account that intelligent actors actively "construct" the concepts and cognitive categories they use for world representation. The old and venerable assumption that the structure of the real world is "mirrored" in the process of cognitive development is simply not tenable any more. All different scientific disciplines that deal with the subject of learning agree that learning is a constructive process, performed by the individual learner in an active manner (in the case of neurobiology, cf. [10]). Each learning model has to consider this fact.

Because it is not possible to capture all important cognitive processes at once in a single model, we decided to concentrate on the modeling of concept formation (including the construction of semantical networks) as one of the most important cognitive abilities (cf. [11]). As there is no universal agreement on a definition of concepts, we use the term as a cognitive representation of a category. *Category* can be defined as a set of things, which have certain characteristics in common, and a *concept* as the information that an individual has about this set [12, 13].[4] At the actual state of developing the model the following cognitive operations are implemented into the system.

1. Supervised *versus* unsupervised learning.
2. Creation of new concepts.
3. Learning in a social milieu.

1. Supervised *versus* unsupervised learning

Supervised learning on the one hand means that the learner gets an immediate response (valuation) after having performed a learning process, for example, solving a problem. In particular the learning system often gets information about the size of the error the system has generated if the solution of the problem was not correct. The well-known valuating system at school *via* numbers (in Germany) or letters (in the United States) means exactly this: the evaluation of a certain examination with the number "three" means that the difference between the best possible success—number "one"—and the factual result of the learner is of size "two," measured on a scale from 1 to 6.

On the other hand there are learning processes that are not supervised, which means that the cognitive task has to be fulfilled by applying particular schemas that the learner has learned before, or by constructing new ones if the old schemas are not sufficient. The famous con-

[4]Note that this is the famous definition of a "set," first defined by Georg Cantor, the founder of set theory.

cepts from Piaget of "accommodation" and "assimilation" mean just such processes. Usually these processes are done without immediate responses or valuations respectively by the environment. A classical example, which is contained in the model, is the construction of semantic networks (cf. [11]).

2. Creating new concepts by analogies

Each cognitive system has to not only learn concepts, including solutions of problems by others, but it also often has the problem of creating new concepts on its own. This creative operation is of course not arbitrarily done but mainly by formation of analogy. In other words, if a learner has to create new concepts by itself—without supervision—it will rather often do so by applying the logic learned before when learning concepts from others [9, 14]. By *analogy* we mean here the perception of a structural equality in two different objects or problems respectively, although these objects are different in content. For example, if a child has learned that five apples plus three apples are equal to eight apples then the child will also obtain that five cherries and three cherries are equal to eight cherries, despite the difference between an apple and a cherry. The child sees the structural equality of the two problems of adding apples and cherries respectively.

3. Learning in a social milieu

The interdependency between individual learning and the social milieu must additionally be taken into consideration when analyzing cognitive ontogenesis, as we mentioned before. Depending on the social environment, a child, for example, can learn more or less different concepts and the names of them according to the stimuli given by their environment. It is rather evident that a child on a farm learns other concepts about animals and plants than a child in a big city. The latter may know animals only as dogs and cats and perhaps some others from a zoo.

It has often been demonstrated that not only the particular concepts a learner has to acquire depend on the social milieu, but also the velocity of the learning process. We need only to consider the famous comparative studies Piaget did in this respect about the different developmental velocities of children in Geneva and children in an Egyptian village. The latter ones were of the same age as the Swiss children but in an earlier stage of development, that is, not as far developed as their Swiss counterparts.

For the analysis of cognitive ontogenesis we [15] decided to undertake the modeling with NNs for several theoretical reasons, of which the most important one is that we are interested in a general theory of meaning and communication [16]. After all, NNs were explicitly developed as artificial models for learning processes, according to the paradigm of the human brain. A lot of operations, which we call "intelligent" are done rather unconsciously without using symbols explicitly; therefore a subsymbolic approach that can lead to symbolic operations is more general than a purely symbolic one.[5]

An individual learner in the model consists of several NNs of two different types. The first type, which is used for the modeling of supervised learning and generating new concepts of different order, is a so-called bidirectional associative network (BAM). The second type that is used to model the generation of semantic networks is a Kohonen feature map (KFM), which is able to learn in an unsupervised way, and which we already described in the preceding section.

The technical details of the different NNs cannot be described here but only the particular potentials of them (cf. [15]). Modeling concept learning with BAM means that we may model learning in a "bidirectional" manner. A concept is usually defined by certain characteristics; for example, a "cat" is defined by "small," "furry," "four legs," "tail," "meowing," and so on. In this case the characteristics are learned by sensual perception. If a child learns to associate the concept cat with the respective characteristics, then the child on the one hand will be able to remember the concept cat if it sees an object with the named characteristics. On the other hand, if the child hears the concept cat, then it will remember the characteristics of a cat. Bidirectional thus means that learning and remembering can be done in both directions, which is exactly what BAM nets perform.

In particular we are able to simulate different forms of perception. For example, we may model artificial actors with different capabilities of perception. Technically speaking, the BAM gets pairs of vectors $(X; Y)$; the X-vectors contain the features and the Y-vectors the concepts for the features respectively.

The task of the BAM nets of an individual learner is to learn different sets of features or characteristics respectively and the association of these sets with their concepts. In the most usual way of these learning processes the learner gets the set of characteristics from their environment and obtains the concepts for these from their (social) environment. In other words, the BAM nets simulate the sensual perception of certain features and learn the corresponding concepts from those artificial actors that are more experienced, that is, who already know the correct concepts.

[5] The distinction between symbolic and subsymbolic AI mainly means the distinction between subsymbolically operating systems like NNs and symbolic systems like expert systems (ES). ES operate with certain symbols and symbol connecting rules. This distinction is not always valid because NNs may also operate with symbols. But for the sake of clearness we keep this conceptual distinction.

Technically speaking, a BAM net operates *via* the construction of a "generation matrix" as the connection between the X- and Y-vectors respectively. For details see any book on NNs. As with each learning NN, a BAM net has a learning rule at its disposal, which is similar to the well-known Hebbian type rules:

$$w_{ij} = \sum_{p=1}^{p} = \sum_{p=1}^{p} x_i^p y_j^p \qquad w_{ij} = Y_i X_j = X_i Y_j. \tag{5}$$

The whole matrix is generated as follows:

$$\begin{aligned} &\underline{\underline{W}} = \sum_{p} = 1, \ldots, p(y_p x_p^T) \text{ with } w_{ij} = Y_i X_j = X_i Y_j \\ &\underline{\underline{W}} = \underline{Y}_1 \underline{X}_1^T + \underline{Y}_2 \underline{X}_2^T + \underline{Y}_3 \underline{X}_3^T. \end{aligned} \tag{6}$$

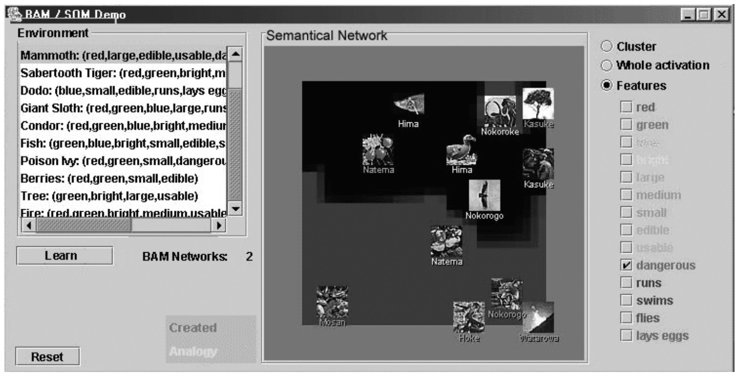
Yet the task done by the BAM nets is not enough because the artificial learner just has an unordered set of concepts, including the respective features, at its disposal. Learning also includes the necessity of ordering the learned concepts, that is, to systematize them and define stronger or weaker connections between them. For example, if a child learns the concepts of "cat," "dog," "bird," and "horse," it also has to learn that the concepts of the three mammals belong "more" together than one of these concepts belongs to that of "fish." The connections in this semantical network between the concepts of the mammals are stronger than the connections between them and the concept fish.

A KFM is the best-known example of unsupervised learning. Its task is collecting and ordering singular concepts, that is, the formation of concept clusters. In our little example one cluster would consist of the three mammal concepts and the concept fish would form another single cluster. Learning occurs in this type conforming to equation (4).

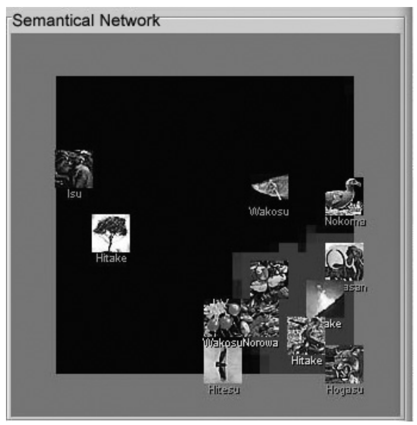
The resulting ensemble of clusters is a formal representation of a semantic network. In our model the KFM gets the information directly from the different BAM networks. The Y-vectors represent the concepts that shall be clustered according to the X-vectors, which consist of the features. Because the KFM clusters only the concepts and not the features it is not always evident why the clusters are generated this way, but this fact can be observed in human cognitive processes as well. It is sometimes not possible to understand the associations made by particular persons because they have formed other concept clusters than one would normally expect.

The result of one run of a KFM depends, simply speaking, with which concept the KFM starts its ordering operations (Figure 2). That is not unrealistic because the order in which concepts have to be learned can also play a large role in the processes of human learning.

The social environment of the artificial learners is modeled with another type of mathematical model. The program operates with CA to



(a)



(b)

Figure 2. Results of the ordering operations of a KFM. (a) An actor created six concepts and constructed six concepts *via* analogy. (b) A second actor created eight concepts and constructed four *via* analogy. The two actors ordered the concepts in different clusters.

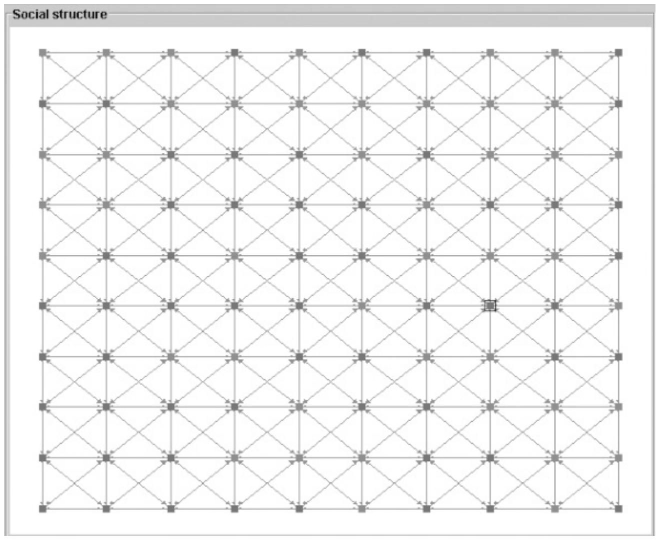
model the social environment. Every cell of the CA represents an artificial actor and accordingly the cells of the CA consist of the different NNs described. The artificial learners interact and learn from one another *via* the logic of the CA. The geometry of the CA is defined by a Moore neighborhood, which means that an artificial learner interacts only with the cells of its neighborhood. "Learning" means that in the social milieu a learner gets new concepts from "older" cells with more experience, which means that the learner is able to associate certain features with the concepts, and the construction of new concepts. In addition, the "primordial" actors, that is, the first ones at the start of the program, are able to create the first concepts by themselves, with which the whole process of cognitive development starts. This model allows us to analyze processes of social learning that depend on the social structure.

In other words, the whole model may be seen as a multi-agent system (MAS) where each agent is represented by the combination of several BAMs and a particular KFM (Ritter–Kohonen type) and where the social structural relations between the learning agents are modeled by CA. The cells of the CA are, therefore, no simple finite state automata—as is usually the case with CA—but complex dynamical systems themselves. In this way we obtain a twofold dynamical system: on the one hand cognitive development occurs as a cognitive dynamic depending on the context, namely, the other learning agents. On the other hand the other agents and the relations between them are influenced by the individual learning processes and in that way generate a social dynamic. The complete model is described in more detail in Klüver *et al.* [17].

According to the descriptions given the model was implemented in a computer program. At first, the BAM networks learned 12 different pairs of X- and Y-vectors. The features and concepts are transferred to the KFM.

The experimental design shown here is based on an initial state that consists of different actors in different stages of development. The "knowledge" that is given to all actors is the same, although not represented in the same order. We wanted to see if there are differences and/or similarities in the "inner structure" of the artificial actors, who construct semantical networks. Also important was the question: Do the actors construct new concepts per analogies? In this case the actor gets only features, because none in its neighborhood know the correct concept, and then has the task of constructing a new concept, if it has already learned some concepts. The construction of new concepts is done in the following way: the BAM nets get vectors with only X-components that are new. The BAM nets compare this new X-vector with all X-vectors the nets have learned so far. The program selects the X-vector that is most similar to the new one, takes the corresponding weight matrix of this X-vector, and constructs a new Y-vector.

In the initial phases the social structure, as we mentioned, is modeled *via* CA, which means in particular that the social relations between the actors are symmetrical. Yet the model also contains processes of dying and creating new actors. As a consequence, in later phases of the simulation runs, there will be older and younger actors with the effect that the younger ones learn from the older actors, but not *vice versa*. By these processes the social relations become asymmetrical, that is, an older actor A influences the younger actor B but not inversely. In our model this changing of the social structure is represented by transforming the CA into boolean networks (BNs). BNs are logical networks where the connections between the cells are defined by logical functions. Figures 3 and 4 show such a process.



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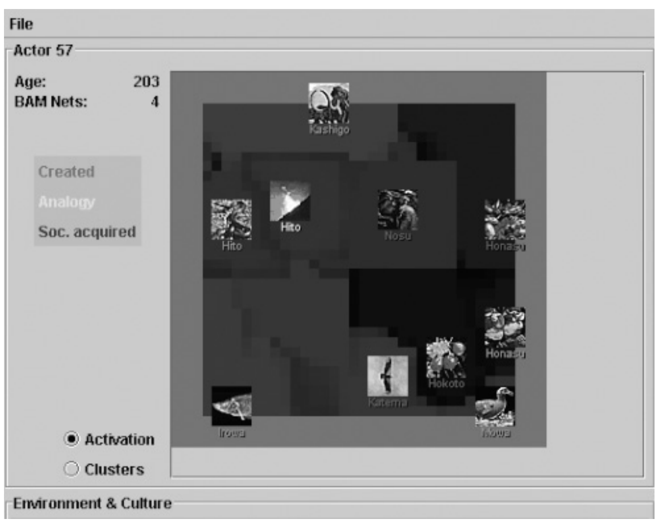
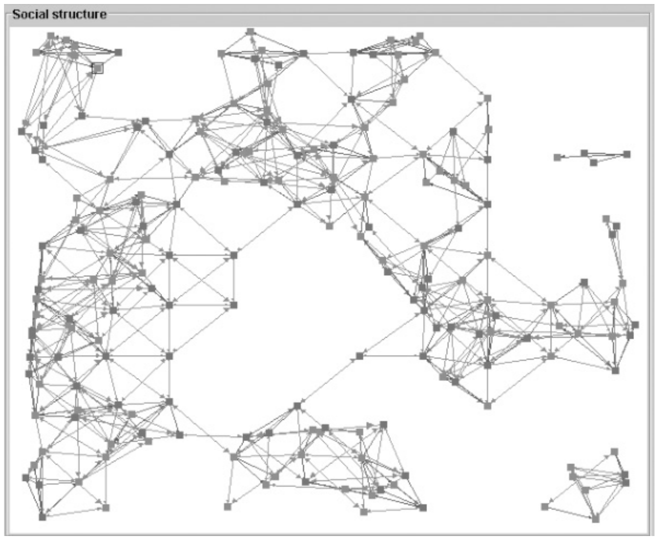


(b)

Figure 3. The initial state at the beginning. (a) The social structure is represented by CA. (b) One actor that has created four concepts, constructed one concept *via* analogy, and learned four concepts from its social environment.



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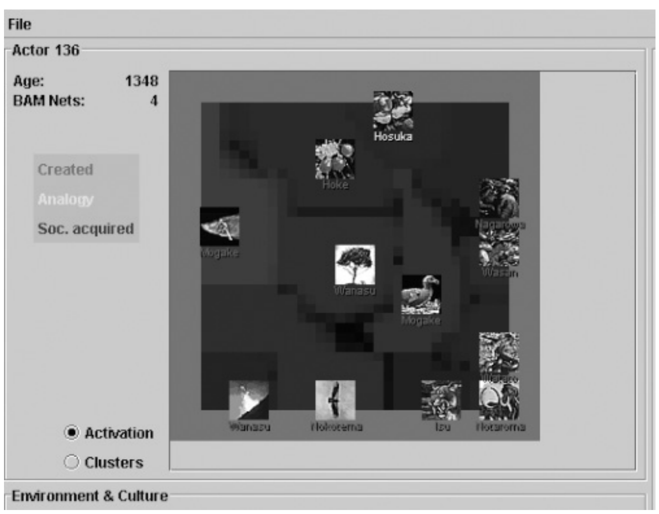

(b)

Figure 4. (a) Examples of transforming CA into BNs. (b) The actor has only one concept constructed *via* analogy, all other concepts are learned socially.

Conclusion

The main goal of this section is to describe the model as a possibility for analyzing different problems of cognitive ontogenesis in a precise manner. This is done by representing important cognitive processes in dependency of social environment *via* the described techniques. The whole potential of the program is still under investigation, which is why we only give some preliminary results. But at least two interesting results can be mentioned.

1. The differences of individual developments are often, although not always, due to the temporal order in which learners get acquainted with new concepts. It is well known that the KFM is sensitive in regard to different initial states, that is, the successive order in which different concepts are presented. But as one can see by the model, this temporal order is an effect of different environments. Therefore it is not enough to analyze the difference of learning milieus in terms of the number of concepts they offer to the learners but it is nearly as important to observe the temporal order of informational processes. In this sense culture as ordered sets of concepts must be taken into regard when analyzing learning processes. The specific culture in which a child grows up has a very important impact on their learning biography. This is certainly a truism, but one that can be studied for the first time in a precise manner by performing computer experiments with different parameters.
2. A social milieu that forces the learner to learn everything the social environment offers can be counterproductive for the learner who has to spend all their time to take over knowledge already known and cannot unfold their own innovative capability—in our model the creation of new concepts. The result will be a static culture, in which no new cognitive achievements are possible. Therefore a cognitive development that allows the learner to unfold creatively must rely upon an environment that allows for "social forgetting," that is, ignoring some knowledge that has been achieved by elders.

Further results, in particular the change of social structure and culture as an effect of innovative cognitive developments, will be published in due time.

4. Final remarks

The goal of this paper was to demonstrate how useful the construction of a hybrid system may be for very different purposes. To be sure, many important problems can be analyzed by the construction of nonhybrid systems like cellular automata (CA). Yet the particular complexity of social-cognitive processes is frequently better modeled by hybrid systems of the kind we just described. Neural CA, which are the subject of this paper, are of course only one possibility among many others. Vertically coupled hybrid systems are, for example, a very suitable approach to model adaptive processes. Therefore, the systematic construction and analysis of hybrid systems is an important task for the analysis of complex systems.

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