Finite Automata, Random Fields, Quantum Fluctuations, and Bell Inequalities

Peter Morgan, Yale University
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Small-scale physics is probabilistic/statistical.

In the presence of significant thermal or quantum fluctuations, a probabilistic approach is needed.

NKS Chapter 6 looks for patterns in the evolution of randomly chosen single instances of an underlying system.

A *random field* is an indexed set of random variables. A lattice as an index set is appropriate for a probabilistic approach to finite automata.

A random field deals with evolution of probabilities directly, instead of with an underlying classical evolution.

The NKS community does not make enough contact with the Bell inequalities argument, which is probabilistic.

Some of the resistance of the Physics community to NKS is because finite automata appear to satisfy the assumptions required to derive Bell inequalities.

Three necessities:

- an NKS approach must have a non-trivial concept of a particle that is not point-like.
- an NKS approach must have a contextual measurement model (best to model experiments instead of measurements).
- an NKS approach must model quantum and thermal fluctuations.

Quantum fluctuations for the Klein-Gordon field

The Klein-Gordon field is differentiable and satisfies the differential equation $\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\phi}{\partial z^2} + m^2\phi = 0$.

My work is with *continuous random fields*, which are closely related to quantum fields (not with an infinitesimal lattice as index set, but sort of).

There is a quantized Klein-Gordon field, and there is also a classical Klein-Gordon *random* field.

A continuous random field is *not* a continuous field, there is a probabilistic fractal structure all the way down.

I have no ontological commitment to continuity. Continuity is just mathematically useful.

For the Klein-Gordon random field, interactions and renormalization are not dealt with (so physicists are not much impressed).

The Gibbs probability density for the equilibrium state of the classical Klein-Gordon random field at temperature T:

$$\begin{aligned}\rho_C[\Phi_t] &\overset{\mathrm{N}}{=} e^{-\beta H[\Phi_t]}\\ &= \exp\left[-\frac{1}{\mathsf{k_B}T}\int \tilde{\Phi}_t^*(\mathbf{k})\tfrac{1}{2}(\mathbf{k}^2+m^2)\tilde{\Phi}_t(\mathbf{k})\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]\end{aligned}$$

The probability density for the vacuum state of the quantized Klein-Gordon field (on any hyperplane):

$$\rho_0[\Phi_t] \overset{\mathrm{N}}{=} \exp\left[-\frac{1}{\hbar}\int \tilde{\Phi}_t^*(\mathbf{k})\sqrt{\mathbf{k}^2+m^2}\,\tilde{\Phi}_t(\mathbf{k})\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]$$

Four changes to obtain the quantum case:

- there's a square root;
- $\mathsf{k_B}T$ becomes Planck's constant (with action units instead of energy units);
- the Lorentz symmetry (the classical Klein-Gordon dynamics is Lorentz invariant, but the Gibbs probability density as an initial condition is not).
- there is also a restriction to positive frequency in the quantum vacuum state.

The Gibbs probability density for the equilibrium state of the classical Klein-Gordon random field at temperature T:

$$\begin{aligned}\rho_C[\Phi_t] &\stackrel{\mathrm{N}}{=} e^{-\beta H[\Phi_t]} \\ &= \exp\left[-\frac{1}{\mathsf{k_B}T}\int \tilde{\Phi}_t^*(\mathbf{k})\tfrac{1}{2}\big(\mathbf{k}^2+m^2\big)\tilde{\Phi}_t(\mathbf{k})\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]\end{aligned}$$

The probability density for the vacuum state of the quantized Klein-Gordon field (on any hyperplane):

$$\rho_0[\Phi_t] \stackrel{\mathrm{N}}{=} \exp\left[-\frac{1}{\hbar}\int \tilde{\Phi}_t^*(\mathbf{k})\sqrt{\mathbf{k}^2+m^2}\,\tilde{\Phi}_t(\mathbf{k})\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]$$

The equilibrium state probability density of the quantized Klein-Gordon field at temperature T:

$$\rho_T[\Phi_t] \stackrel{\mathrm{N}}{=}$$

$$\exp\left[-\frac{1}{\hbar}\int \tanh\left(\frac{\hbar\sqrt{\mathbf{k}^2+m^2}}{2\mathsf{k_B}T}\right)\tilde{\Phi}_t^*(\mathbf{k})\sqrt{\mathbf{k}^2+m^2}\,\tilde{\Phi}_t(\mathbf{k})\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]$$

There is a difference between quantum fluctuations and thermal fluctuations, and this is the best characterization of what the difference is (that I know of).

Bell inequalities for random fields

Bell inequalities can be derived for non-contextual particle property models.

Bell inequalities cannot be derived for random fields that have thermal or quantum fluctuations.

I argue this at length in *J. Phys.* **A 39** (2006) 7441-7455.

We can use four random variables to model an experiment that violates a Bell inequality:

s_A and A are measurement settings and measurement results associated with a space-time region $\mathcal{A}$.

s_B and B are measurement settings and measurement results associated with a space-time region $\mathcal{B}$.

In fact, we only have statistics, but we take probabilities to be good models for statistics.

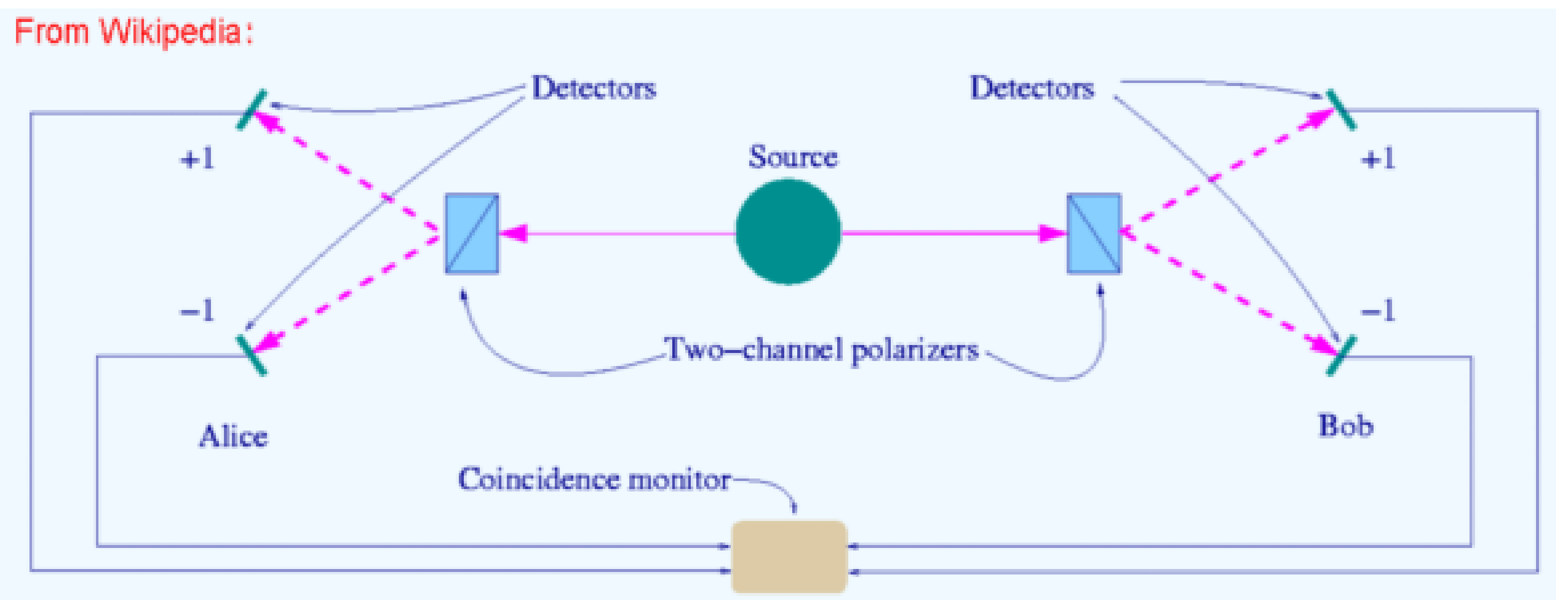
time	t_1	t_2	$t_2+\epsilon$	t_3	t_4	t_5	$\cdots$
place	$\mathcal{A}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{B}$	$\cdots$
measurement setting s_A or s_B	a_2	a_1	b_2	a_1	b_2	b_1	$\cdots$
measurement result A or B	$+$	$-$	$+$	$+$	$-$	$+$	$\cdots$

The experimental data we have is a list of events: the times and places at which the events happened, the settings, and the results.

In general we have control neither of when measurement events happen nor of the measurement settings.

This is still idealized, but less so than a probability distribution.

time	t_1	t_2	$t_2+\epsilon$	t_3	t_4	t_5	$\cdots$
place	$\mathcal{A}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{B}$	$\cdots$
measurement setting s_A or s_B	a_2	a_1	b_2	a_1	b_2	b_1	$\cdots$
measurement result A or B	$+$	$-$	$+$	$+$	$-$	$+$	$\cdots$

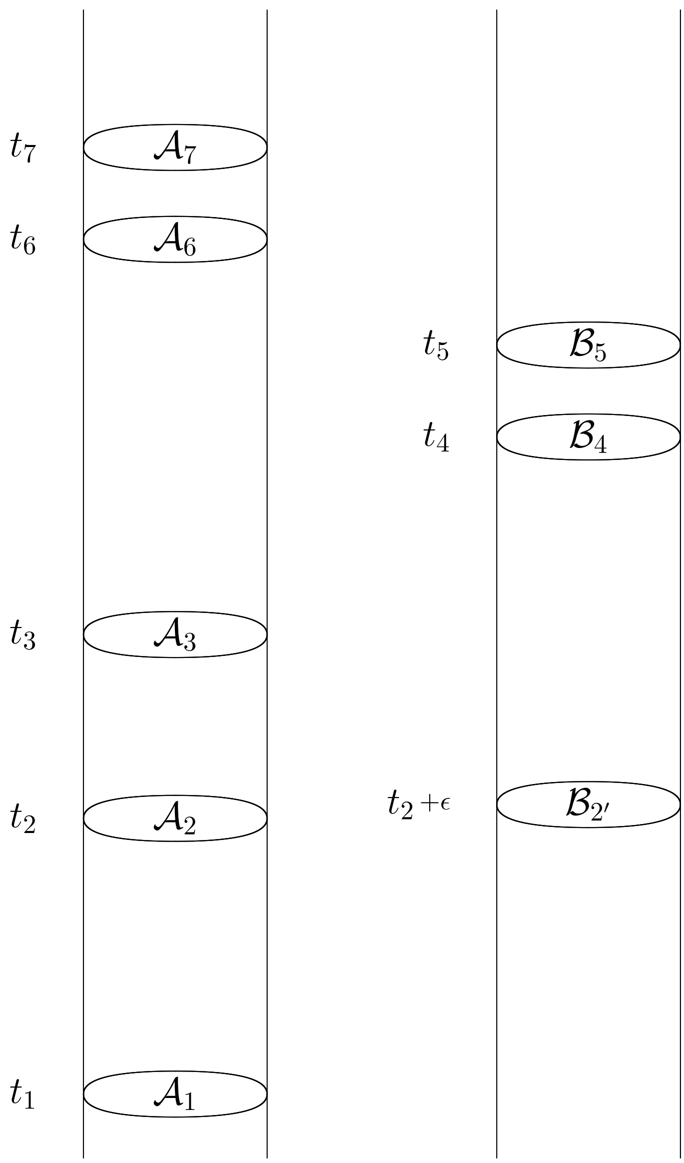


The source sends pairs of "photons", one in each direction.

Each photon encounters a two-channel polariser whose orientation ($s_A = a_1$ or a_2) can be set by the experimenter.

Signals from each channel are detected and coincidences of four types ($++$, $--$, $+-$ and $-+$) are counted.

time	t_1	t_2	$t_2+\epsilon$	t_3	t_4	t_5	$\cdots$
place	$\mathcal{A}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{B}$	$\cdots$
measurement setting s_A or s_B	a_2	a_1	b_2	a_1	b_2	b_1	$\cdots$
measurement result A or B	$+$	$-$	$+$	$+$	$-$	$+$	$\cdots$



time	t_1	t_2	$t_2{+\epsilon}$	t_3	t_4	t_5	$\cdots$
place	$\mathcal{A}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{B}$	$\cdots$
measurement setting s_A or s_B	a_2	a_1	b_2	a_1	b_2	b_1	$\cdots$
measurement result A or B	$+$	$-$	$+$	$+$	$-$	$+$	$\cdots$

We find pairs of events at nearly matching times in regions $\mathcal{A}$ and $\mathcal{B}$, to construct a list of "simultaneous" events.

time	t_2	t_9	t_{12}	$\cdots$
s_A	a_1	a_1	a_2	$\cdots$
A	$-$	$-$	$+$	$\cdots$
s_B	b_2	b_1	b_1	$\cdots$
B	$+$	$-$	$+$	$\cdots$

Taking the ergodic hypothesis to justify taking the data at t_2, t_9, t_{12}, ... to be an ensemble of independent events, and

ignoring the times, we can model the statistics of this data by a probability distribution $p(s_A, A, s_B, B)$.

The probability distribution $p(s_A, A, s_B, B)$ is just part of an initial condition, at the time of the experiment, for a random field model.

Initial conditions are not constrained by classical physics, but more experimental effort is needed to set up unlikely initial conditions, which have higher free energy.

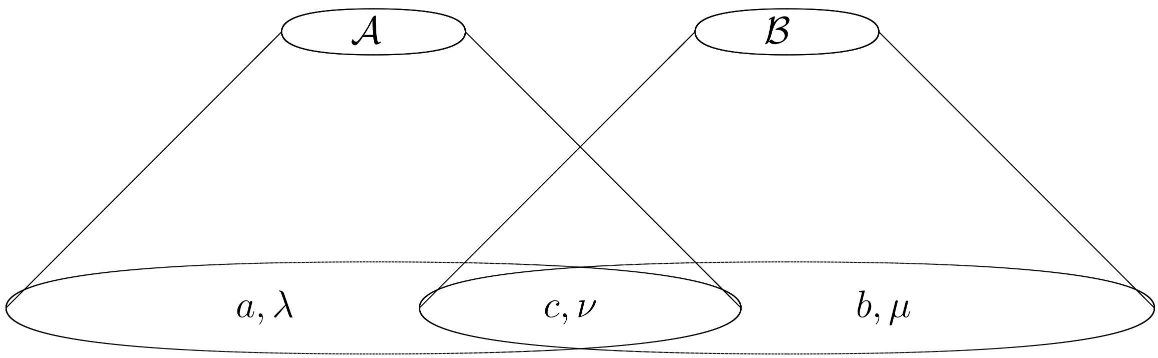
Experiments that violate Bell inequalities are not easy.

The experimentally grounded $p(s_A, A, s_B, B)$ at the time of the experiment (partially) determines what the initial conditions must have been in the past and will be in the future.

What Bell says is that the field in the past cannot have been what it would have had to have been to result in what the field is now, because that would be weird.

Saying that weird initial conditions in the past are OK is known as the "conspiracy loophole".

To derive Bell inequalities, Bell introduces various *a priori* constraints on what the initial conditions in the past are allowed to have been.



Bell's constraints are based on an idea of common cause that is well-founded for a classical particle model, but is relatively unmotivated for a random field.

For a classical particle model, two particles come from a single source, the common cause of two correlated events.

The correlations of a random field (and of a quantum field) evolve from correlations at earlier times.

There is a "distributed" cause, not a common cause.

Correlations now evolve out of past correlations and into future correlations.

For example, there is equilibrium now because there was equilibrium before.

There are non-local correlations between observables of a classical field at thermal equilibrium, enough to make it impossible to derive Bell inequalities.

Where do equilibrium correlations come from? They're just there (or we arranged it).

The violation of Bell inequalities shows that there are no simply localized particles that would imply pervasive common causes.

There is a distinct tradition of discussing Bell inequalities as about measurement incompatibility, or as showing that particle properties must be *contextual*, not as about nonlocality,

starting with Accardi (1981) and Fine (1982).

From the experimental data, we can construct probability distributions $p(A, B|a_1, b_1)$, $p(A, B|a_2, b_1)$, $p(A, B|a_2, b_2)$, and $p(A, B|a_2, b_1)$.

Without making any assumptions about locality, we can prove that we cannot in general construct a probability distribution $p(A_1, A_2, B_1, B_2|a_1, a_2, b_1, b_2)$ that has all four of the above as marginals.

We cannot talk about measuring both a_1 and a_2 at the same time.

The Kochen-Specker paradox says the same thing.

When we construct an NKS or random field model, we must ensure that there are no well-defined particle properties.

The experimental data idealized as a probability distribution $p(s_A, A, s_B, B)$ is a (partial) initial condition for a quantum field model as much as it is for a random field model.

A quantum field state determined by this experimental data determines correlations at future and past times to the same extent as a classical random field.

So there is as much "conspiracy" in a quantum field model as there is in a continuous random field model.

We should not conclude that quantum field theory is unreasonable, but that a continuous random field model is as reasonable.

Measurement for random fields

Ideal classical measurements do not affect other measurements or the system they measure.

If the quantum fluctuations of a measurement device are significant, we model the measurement device and its quantum interactions with the "measured system" explicitly.

Equally, when we construct a quantum field model, if the thermal fluctuations of a measurement device are significant, we model the measurement device and its thermal interactions with the "measured system" explicitly.

A random field model is not much more difficult than a quantum field model if thermal fluctuations already have to be taken into account.

Even if we cannot reduce quantum fluctuations, we can *imagine* what results we would obtain if we could.

A quantum model is intrinsically contextual, through measurement incompatibility, whereas a random field has to be explicitly contextual.

We have to include a description of experimental apparatus and the effects of its thermal and quantum fluctuations in our random field models.

A random field model is contextual insofar as it includes apparatus degrees of freedom.

This is not contextuality in the usually pejorative sense that particle properties depend on what apparatus is used.

There are no precise particle properties.

Discrete event measurements

A "discrete event measurement device" is a metastable thermodynamic state that is tuned *not* to transition to its registration state (except for its dark rate statistics).

e.g. CCDs, photographic plates, semiconductor devices, bubble chambers.

When a transition to the registration state happens, a feedback process returns the device to its metastable state as quickly as possible (unless it's a photographic plate; a bubble chamber is returned to its metastable state cyclically rather than through feedback).

When the device is put near various plugged-in and turned-on apparatuses, different statistics for transitions to its registration state are observed — a change of the environment changes the response.

Discrete transition events are a consequence of (engineered) thermodynamic properties more than of any discrete structure of the external field.

The discrete events do not have to be taken to represent the arrival of individual particles.

Conclusion

NKS modelling should be probabilistic, and include something like quantum fluctuations, distinct from thermal fluctuations.

If it's a classical model, it's a random field model.

NKS modelling should include thermodynamic transitions.

NKS modelling should have different effective evolutions at different scales and in different thermodynamic conditions.

The effective dynamics should in some conditions lead to identifiable particles, in other conditions not.

NKS modelling should be more formally cognizant of *constraints*.